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AN EVALUATION OF IMAGE ANALYSIS TECHNIQUES AS
APPLIED TO THERMOGRAMS USED IN BREAST CANCER DETECTION.

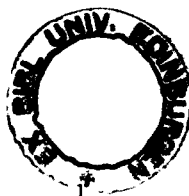
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ABSTRACT OF THESIS

Due to the lack of a formal theory for designing image analysis systems it has been necessary to develop and implement a package of computer programs for evaluating image analysis techniques and designing image analysis systems. The package allows the image analysis system designer to experiment with different algorithms and techniques on his data. This package is described and a classification of image analysis techniques is given. Finally the package and a class of image analysis algorithms are applied to a particular problem: 'the computer assisted interpretation of breast thermograms for use in the early detection of breast cancer'. It is an important problem to due the observation that breast cancer increases surface temperature and that its early detection increases the five year survival rate of sufferers. Thermography is somewhat nonspecific but may be effectively used as a screening technique. An assessment is made of the algorithms applied to breast thermograms and proposals for future research are given.

ACKNOWLEDGEMENTS.

The author expresses his gratitude to his thesis supervisor, DR. J P. Gray of the Department of Computer Science, University of Edinburgh, for his helpful ideas and encouragement.

The author would also like to thank Professor E. Samuel and DR. A. Scott of the Department of Diagnostic Radiology, University of Edinburgh, for discussions on thermography and access to the patient data used in this research.

Thanks should also go to Mr. J L. Watson of the Atomic Weapons Research Establishment for discussions and information on the technical aspects of the patient data and the thermographic scanner.

Thanks are also extended to my wife, Alexis, for her patience.

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1. INTRODUCTION.

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1.1 BACKGROUND.

Originally proposals were put forward by the author to automate certain aspects of the interpretation of breast thermograms as used in the early detection of breast cancer, and in particular to automate the computer 'mapping' of the outline of the breast and the subsequent derivation of clinically meaningful data, i.e. the temperature distribution within the breast outline. The research described in this thesis was carried out because of the lack of a theoretical approach for designing such an image analysis system. In fact, no clear theoretical guidelines are available for designing an image analyser in an arbitrary problem domain. A package of computer programs is described which makes available a large number of techniques developed for image analysis. These techniques are sufficiently general that they can be applied to many different types of image. The researcher can add commands and algorithms to the

package as new techniques are developed. The use of the package for evaluating techniques applicable to an image analysis system is demonstrated by examining breast thermograms; these have been found to be clinically useful in the early detection of breast cancer.

Since the introduction of the digital computer, there has been a constant effort to increase the number of computer applications. Some of the motivation for this effort comes from important practical problems which require more efficient and more accurate solutions. It also comes from the challenge of programming a computer to solve problems never attempted by computer before. Similarly, the development of image analysis has been motivated by both practical and theoretical considerations. At present, the ability of computers to perceive and analyse their environment is very limited indeed. A number of different devices are available for converting light, sound, temperature, etc. to electrical signals. When the environment that a computer perceives is very carefully controlled and the electrical signals have a simple interpretation (as is the case with standard computer input devices), the problems of perception and analysis become relatively trivial. However, as we move from making a computer read punched cards or magnetic tapes to making it understand hand-printed characters or medical images, we move from problems of sensing the data to the much more difficult problems of interpreting the data.

The apparent ease with which vertebrates and even insects perform perceptual tasks is at the same time encouraging and frustrating. Psychological and physiological studies have given us a great many interesting facts about animal perception, but an insufficient understanding for us to duplicate animal performance with a computer. The problem area has a certain unique fascination in

that perception is something everyone experiences but no one really understands. Paradoxically, we are all expert at perception, but none of us knows much about it.

The lack of a complete theory of perception should not prevent us from attempting to solve modest problems in image analysis and perception. A general specification of image analysis is the production of computer-based methods for simplifying and describing images in order to answer such questions as, 'Does the image contain a particular object?', 'Name all the objects in the image?', or more generally, 'Describe the image?'. This is a much more ambitious specification of the problem than is dealt with here. This thesis pursues a method of combining computer techniques and speed with human judgement and analysis, such that a package is used to select 'appropriate' image analysis techniques for problem 'specific' data.

These are reasons why computers have been used to perceive and analyse images, but why should we presume that computers are needed to assist in the diagnosis of disease? The accuracy of diagnosis is inevitably related to the consideration of all possible diseases which may explain a set of symptoms and findings. A consideration of each possible disease in the light of the established symptoms and findings is necessary to determine which one has the greatest probability of being correct. Physicians have been performing these mental gymnastics with varying degrees of effectiveness since man assumed responsibility for the medical care of his kind. When the range of diagnoses or the number of recognised symptoms increases to such an extent that they can no longer be handled efficiently, physicians respond by specialisation and in this way limit the number of diseases to a range which can be comfortably managed. We are fast approaching the limit to which we can specialise and still continue

to meet the needs of society, as the number of known facts about disease and methods for patient care increase at an unprecedented rate. For this reason an effective use of computers can be made in health care by presenting and analysing relevant facts and thus assisting the physician in his diagnosis.

Turning now to the specific problem dealt with in this thesis, the author initially considered how to make an effective use of computer graphics rather than image analysis in medicine. Image analysis is the computer analysis of given images, whereas computer graphics is the synthesis of images by computer. Image analysis was finally chosen in preference as it seemed to provide more direct and obvious benefits in a practical clinical setting. The greatest use of computer graphics in medicine to date has been in the area of medical research; examples of this are the graphical presentation of data, such as in the physiology laboratory, and computer generated stereoscopic displays of molecules. The literature available (GRO71,WIL70,WHI70) suggests that computer graphics techniques used in medical research cannot be usefully applied in a practical clinical setting. There are only two areas where computer graphics have been successfully used in clinical medicine. They are radiotherapy treatment planning (BEN70), and information systems using visual display units where graphical and textual information are both presented (HSC72). Image analysis on the other hand has a far greater potential application in medicine. This is not surprising: when we consider the work doctors do we see that a large part of it involves examining images and presenting a diagnosis. These images are X-rays, thermograms, chromosome photographs, physiological monitorings, etc. To summarise computer graphics are mainly used for presenting abstractions as images; the reverse

process appears to be what doctors do.

Up until now the bulk of the work that has been done in biological image analysis has been chromosome classification (LED69, RAM69, EVA69), although some work has been done on computer assisted diagnosis of X-rays (LOD63a, LOD63b, HAL71, LEH72, DWY72). The research described here is applied to computer aided thermogram interpretation used in the early detection of breast cancer.

1.2 THE PROBLEM.

Breast carcinoma is the most common cause of female carcinoma deaths. Between 11,000 and 12,000 women die every year in the United Kingdom from this disease, about one in five of all carcinoma deaths. Breast carcinoma is three times more common than any other female carcinoma. In addition it is one hundred times more common in females than in males (REG67).

The importance of early screening so as to increase the five year survival rate has been stressed by many writers (LIL69, ISA72, FRE72). Thermography, a method for producing an infra-red image of the surface of part of the body, is based upon the physical principle that the amount of radiation emitted by an object depends upon its absolute temperature. The image obtained by an infra-red sensitive detector as it scans over a patient represents surface temperature variation. It has been observed that carcinoma increases surface temperature thus making breast thermography a clinically useful tool in the detection of breast malignancy. Research has shown that absolute temperature has no diagnostic value (LAW63). Symmetry and relative temperature are the basis from which

most diagnostic criteria are developed, such as:

- 1) temperature asymmetry of the breasts,
- 2) temperature difference of the breasts,
- 3) the presence of a localised 'hot spot',
- 4) areola temperature difference.

In addition relatively few features are required for the classification of the *breasts* into the normal and abnormal classes. For large scale screening, a great number of these images would need to be examined with consistency. The human visual system is poorly adapted to quantitative brightness estimation and noncontiguous area comparisons (ST072,ZUS70). Thus thermography seems amenable to automation if some of the fundamental image recognition and analysis problems can be overcome.

1.3 CONTEXT.

This section gives a short summary of the uses of images in medical diagnosis, the development of breast thermography and the field of image analysis.

1.3.1 The use of images in medical diagnosis.

The need for computer assistance in the quantitative measurement of images has resulted through developments in physics and electronics in the past few years. These developments have produced much improved measurement techniques in medicine, a point has now been reached where, by various methods, a vast quantity of different types of image can be generated that give a large variety of information about the state of bodily function and structure. Such images include thermograms, gamma camera pictures, brain scans,

X-rays, ultra-sonic pictures and a number of other images associated with the pathology of blood and other cells. In most cases the interpretation of these images is highly subjective and the problems in handling this new found source of information are only just being realised. The magnitude of the problem is such that 'the use of computers' is called for, but this is a simple retort to a complex problem.

As a result there is an increasing need to develop image analysis techniques for medical applications. As the capabilities of computers grow, the automatic processing of image data becomes more appealing and could have a major impact on medicine. Unfortunately there are currently no clear theoretical guide-lines to assist us in our analysis of these images. Most image analysis systems hitherto have been basically a collection of ad hoc techniques which appear effective in their current application. This thesis reports on a package which allows the researcher to evaluate different algorithms on his particular data and then choose the most suitable ones.

1.3.2 Breast thermography.

The physician's diagnostic abilities exceed by far those of current computer technology, especially when visual interpretation is required. The useful role of computers in medical diagnosis in the immediate future is in the assistance of the physician's diagnostic effort, especially in stylised, highly repetitive tasks where the sheer bulk of the workload makes it reasonable to attempt automation. The most difficult task in the meaningful application of computers to medicine is the selection of significant problems which have practical importance and which are amenable to solution using current technology. Breast thermograms are suitable for automated

interpretation because of their relative simplicity compared with other medical images. The diagnostic criteria involve essentially a measure of the symmetry and relative temperature of the breasts. Relatively few features are required. Only rudimentary knowledge of anatomy and pathology is necessary for thermographic interpretation. In thermograms the object is opaque, unlike the overlapping objects present in X-rays. Classification is limited to two classes, normal and abnormal. The image is inherently quantitative in nature due to the direct representation of the physical temperature of the breasts.

The desirability of thermographic screening for breast malignancy and the general utility of a low-cost method for automating this examination are clear. This thesis demonstrates some techniques that show promise for the successful automation of breast thermogram interpretation.

1.3.3 Image analysis.

Image analysis by computer encompasses a wide variety of techniques and mathematical tools. Most of these have been developed in response to three major problems.

1) Image digitisation and coding:

i.e. the conversion of images from continuous to discrete form, and the compression of digitised images so as to conserve storage space or channel capacity if the image is to be transmitted.

2) Image enhancement and restoration:

i.e. the improvement of degraded images where collection or capture of the image may not be possible to repeat. Images from space flights are an example of these types of image. They may be blurred, noisy or distorted by the camera.

3) Image segmentation and description:

i.e. the conversion of images into simplified 'maps', the measurement of properties of images or image parts, and the classification or description of images in terms of parts and properties.

Image segmentation and description is the aim of the majority of the techniques described in this thesis, although some techniques which enhance images are included. Historically image analysis algorithms are normally given for the continuous case, i.e. where the image is considered to be a function. The implementation of these algorithms on a digital computer normally requires a different interpretation of these images, i.e. where the image is considered as an array of points.

1.3.3.1 Images as functions.

Informally, an image is a flat object whose brightness or colour or temperature may vary from point to point. This variation can be represented mathematically by a function of two spatial variables. An image can be represented by a single real-valued function, say $F(x,y)$. The value of this function at a point will be called the grey level or brightness or temperature of the image at

that point. It is customary to assume that functions which represent images are analytically well behaved so that, for example, these functions are integrable, have invertible Fourier transforms, etc. It is usual also to regard these functions as having values that are non-negative and bounded,

$$\text{i.e. } 0 \leq F(x,y) \leq N \text{ for all } x,y.$$

1.3.3.2 Images as arrays.

When an image is digitised, a sampling process is used to extract from the image a discrete set of real numbers (samples), and a quantisation process is then applied to these samples to yield numbers having a discrete set of possible values. In most practical situations, the samples are the values of the image at a discrete, usually regularly spaced, set of points, or (more realistically) averages of the values taken over small neighbourhoods of such points. Such a set of samples can be represented, for computer processing purposes, as a rectangular array of real numbers. The samples are usually quantised to a set of equally spaced grey level values. If the unit of measurement is suitably chosen, these values can be taken to be integers. Thus a digitised image, or digital image, can be regarded as an integer array. The elements of a digital image array are often called picture elements, pixels or pels, or sometimes just points.

1.4 OVERVIEW.

It is important to note that merely naming a problem domain says little about the image analysis problems encountered therein. As an example, consider for a moment optical character recognition,

which is in some ways a rather simple problem domain. The difficulty of a character recognition problem depends upon many things: whether the characters are made by hand or machine; if by machine, whether a single font or many fonts are used; if by hand, whether one author or many will be encountered; and so on through a long list of specifications. Hence image analysis is concerned not only with a wide variety of problem domains but with an enormous variability of problems within a single domain. It would be naive, of course, to expect that a single technique or even a small number of techniques would be applicable across such a broad spectrum of problems. Indeed this variability makes it difficult to characterise image analysis as a whole, except to note that the process is essentially one of simplification. The general problem is to transform an image, represented perhaps by tens of thousands of bits, to a description or classification represented by only tens of bits. Interesting methods are ones which simplify images by suppressing irrelevant detail, characterise shapes and sizes of objects in an image, integrate parts of an image into meaningful entities, and in general reduce the complexity of the data.

This work is concerned with how intensity arrays derived from some sort of camera or transducer can be understood. The work lies in the direction of image analysis rather than scene analysis, but it should be understood that these terms only represent a rough categorisation preferable to the obsolete 'high-level' and 'low-level' terminology previously used. One might think that image analysis would involve inherently little problem solving and that it might therefore be an easier area to work in. However, facing facts, pulling something meaningful from an array of image points is already a problem bigger than most novices imagine. For example edges of

images often generate only very small intensity differences that are hard to deal with because information from cameras and other sensors is noisy and often distorted. Sensors supply some of the degradation and the objects themselves contribute more, in the form of dirt, texture, shadows and multiple reflections. If a line finder is too sensitive, all these problems create a jungle of falsely perceived edges and lines. If the line finder is not sensitive enough, some legitimate lines and edges will be missing from the drawing. Unfortunately these difficulties cannot be handled by careful tuning of sensitivity parameters. Some noise-produced lines are stronger than real ones and cannot be properly disregarded. Filtering theory also lacks power in image analysis because the noise does not have simple properties that underlie the linear filtering approach.

In artificial intelligence the central questions have often had to do with problem solving, and researchers in that field have therefore more strongly pursued the problem solving aspects of vision, leaving some of the more basic image analysis aspects out to pasture. The development of an interactive image analysis system involves both system aspects and computational algorithms for image analysis. Up until now most successful image analysis and classification systems use 'explicit feature selection and classification logic design based on structural analysis through intuition' (KAN72)! The system described here allows the designer to choose interactively the techniques which he can see match his data best. This system is an attempt to solve the 'love at first sight' (PIP68) effect in research where the researcher has little alternative but to continue with and try to improve on his original thesis even though results are not very good. Nevertheless, the author feels strongly that any current development of image analysis

techniques will come about through direct involvement with real data and by application to a practical problem. The research described here was carried out with that in mind.

1.5 SUMMARY OF CHAPTERS.

A brief summary of each chapter and a description of the contents of the appendices is given below.

Chapter 1 is an introduction to the thesis. It describes the background to this research and outlines the particular problem tackled.

Chapter 2 surveys the literature in the fields covered by the thesis: image analysis systems, image analysis techniques, thermography and visual perception.

Chapter 3 classifies different image analysis techniques into general headings, and discusses the theory behind each technique.

Chapter 4 describes the image analysis package BLIPS designed and implemented by the author. Its principal objectives and structure are outlined. The chapter also includes some examples of different algorithms applied to artificial data and an evaluation of the different techniques.

Chapter 5 covers the particular problem dealt with in this thesis, the analysis of breast thermograms. The background to the problem is dealt with in depth and the designing of a system for analysing thermograms is outlined.

Chapter 6 evaluates the different algorithms applied to the thermograms and discusses which techniques appear the most suitable.

Chapter 7 summarises the thesis and proposes areas for future research and development.

Appendix 1 is a user guide to the BLIPS package.

Appendix 2 is a bibliography for the thesis.

2. LITERATURE SURVEY.

2.1 Introduction.

2.2 Image Analysis Systems.

2.3 Image Analysis Techniques.

2.4 Thermography.

2.5 Visual Perception.

2.1 INTRODUCTION.

This chapter reviews the literature currently available on the areas covered by this thesis. The section on image analysis systems deals with complete systems for specific applications, e.g. the identification of human faces by computer (KAN73), and general purpose systems which have been used for analysing a number of types of image. The next section reviews papers on image analysis techniques. As there are literally hundreds of papers on this subject published every year in a wide variety of journals, I have tried to cover only a representative sample of those currently available. The section on thermography deals with a number of aspects including thermographic scanners, breast cancer diagnosis using thermograms, breast cancer screening and breast cancer statistics. The final section surveys some of the important literature on visual perception which is applicable to image analysis and breast thermogram

interpretation.

2.2 IMAGE ANALYSIS SYSTEMS.

Before dealing with any specific system a specification of the general structure of most image analysis systems which have been developed is given. This is not a hard and fast specification where each technique is bound to each stage but a general and flexible grouping of techniques. In some systems two stages are merged, in some others one of the stages is split into two, while in others a stage is entered more than once if a subsequent stage cannot make use of the output from a previous one. However, image analysis and recognition systems can usually be seen to comprise four distinct stages as shown in figure 1.

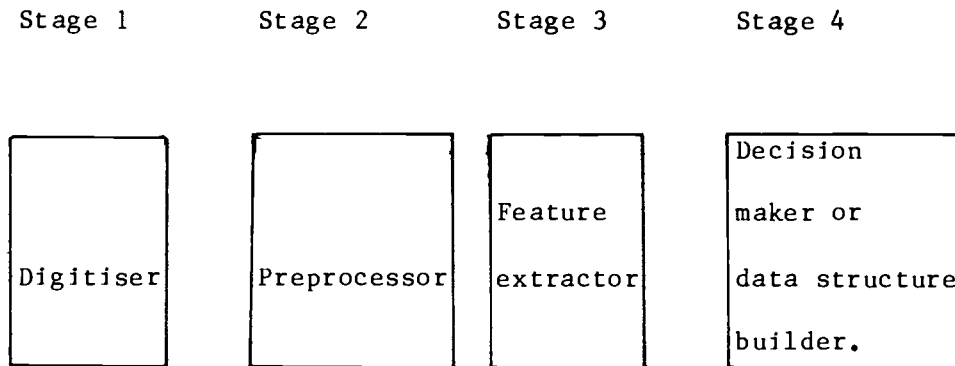


Figure 1.

A hypothetical image analysis or recognition system.

Stage 1: The image digitiser, which converts an analogue image into an array of digital numbers, where each number represents brightness at a particular point.

Stage 2: The preprocessing system, which contains an application

dependent technique for enhancing preselected features and removing irrelevant details.

Stage 3: The feature extraction and selection system, which segments the image and parameterises the segments in preparation for the classifier or data structure builder.

Stage 4. The pattern classifier or data structure builder, which assigns the image to a particular class or produces a data structure for further interrogation or manipulation.

Major progress has been made on the classification problem using a statistical approach (DUD73), or the recently developed syntactic approach (FU70). Significant success has also been achieved building data structures suitable for further manipulation (BAR70). However, research into feature extraction and preprocessing has produced relatively little success as far as generality is concerned. This thesis is primarily concerned with the choice and development of techniques used in stages 2 and 3 shown in figure 1 and described above. The approach adopted has been to provide a package which allows the researcher to iterate between the preprocessing and feature extraction stages until he finds a suitable algorithm and parameters for his problem. The methods currently used in single problem systems and general purpose systems are covered below.

2.2.1 Specific application-orientated systems.

The vast majority of the published papers in the field of image analysis are essentially descriptions of a technique that either the author has invented or has found a novel use for in his application. This results in very few papers describing complete systems. There unfortunately also seems to be very few image analysis systems in

everyday use. This is either because they are not cost effective or they are unable to produce results of a sufficiently high standard or reliability. A rather interesting paper (HAR75) goes into the problem of trying to use a small and relatively cheap computer to check for faults in simple electronic assemblies, e.g. as used in colour televisions. The conclusion that can be drawn from this paper is that to do this type of work in real-time using a minicomputer requires that the image analysis techniques be very basic indeed. Most researchers, especially in the field of artificial intelligence, have had large powerful computers at their disposal and have not allowed the size or efficiency of their systems to deter them.

A number of systems have been described which do try to overcome these problems. A system described by T. Kanade (KAN73) uses a number of small computers to get round the large computer and long elapsed time problems. A number of specialised medical systems have been reported (CHI74,DWY72) which use a dedicated minicomputer to carry out the calculations and do not attempt to respond in real-time. Another technique which has been used is to incorporate an interaction with a human operator to guide or influence the analysis (GOL72).

The future for specialised image analysis systems looks brighter, with the apparent reduction in the cost of computer hardware and the increase in the ease of computer circuit design (which may allow implementation in hardware of some algorithms). The effects of developments in parallel processing systems (DUF73) are currently unclear.

2.2.2 General purpose systems for image analysis.

This type of system represents responses to demands for generality in the experimental domain. The development of interactive image analysis and classification systems involves both computer system aspects and computational algorithms. A recent paper (KAN72) covers the development of these types of system, starting with the era of learning machines and presents reasons for the current emergence, as a general approach to practical pattern recognition problems, of graphics oriented interactive pattern analysis and classification systems. A wide variety of systems and their application to problems in one dimensional and two dimensional patterns are surveyed (SAM68,GOL69,PAT69,GUI68). Various aspects of alternative hardware and software implementations are commented upon and computational algorithms and mappings relevant to interactive analysis and classification of patterns are discussed.

Although they have been called 'systems', only a few actually deserve this name. For the most part they appear to have been developed piecemeal from software tailored to fairly specific applications, without provision for the capabilities which should be incorporated. Systems which allow feedback from man to machine and vice versa are properly viewed as a subset of a larger group which includes interactive systems for mathematics, computer aided design, question answering and computer animation. While each has individual requirements, past developments in these fields have strongly influenced the development of existing systems and will continue to exert such an influence.

We can conclude from this that general interactive image analysis systems have started to come about through responses to

demands for some generality in the experimental domain. They were helped by an improvement in man machine communication, graphics terminals and high data rate communications links. We should see further developments with more improvements in display technology e.g. the availability of matrix plotters, video output and large virtual memory machines.

2.3 IMAGE ANALYSIS TECHNIQUES.

The development of techniques for the computer analysis of images and scenes began over twenty years ago. Most of the work in this field has been application oriented. Some of the major application areas are automation (robot vision), cytology, radiology, high-energy physics, remote sensing, and document processing (character recognition). However, many of the techniques and algorithms, even if developed for a particular application, can be used in other areas. One of the things this thesis attempts to do is to group and classify similar techniques so that an understanding of the principles behind the techniques can be found.

The goal of image or scene analysis is the extraction of a description from the given image. This description may consist of a set of numerical data (feature measurements), or it may be some type of data structure which represents relationships between significant parts (segments) of the image, as well as properties of these parts. Thus, in general, image analysis involves segmentation of the image into parts, measurement of properties of the parts (including grey scale dependent properties such as texture, and geometrical properties such as size and shape), and the determination of relationships among the parts. In addition it may be desirable to

'preprocess' the image, i.e. to modify it so as to make the subsequent analysis steps easier and more reliable.

Many papers on image analysis appear each year in a large number of different journals, few of which are primarily concerned with image analysis. Fortunately for the newcomer to the subject A. Rosenfeld produces a survey of the papers published every year or so (ROS69,ROS72,ROS73,ROS74,ROS75). Some of the topics covered in this thesis: digitisation, differentiation, averaging, template matching, region analysis, contour following, enhancement, transformations, line fitting, line description, and shape description have been dealt with in many papers. Below I present a brief summary of each of these techniques and a selection of representative papers.

2.3.1 Digitisation.

Digitisation is the transformation of an analogue, i.e. continuous, signal to a discrete numerical representation. The two primary parameters in digitisation are the quantisation range, i.e. the range of grey levels, and the sampling rate, i.e. the size of each pixel. Both of these parameters effect the subsequent fidelity of the image. The theoretical basis for choosing our quantisation levels and sampling rate can be found in Shannon's classic paper on information theory (SHA49). The Fourier transform (BRA68,LIN66) of digitised images has been studied to give an insight into some theoretical aspects of digitisation. If an image's transform contains no high frequencies then the image itself cannot contain abrupt changes in grey level, so we need not digitise such an image so finely.

A number of theoretical papers have been published on image fidelity (BAL40,BUD72), mostly concerned with the transmission of digital television pictures. The types of problem which occur in one dimensional signal digitisation, e.g. aliasing, have also been dealt with for two dimensional signals (LEG73). A important paper (PET62) generalises the digitisation of signals to multidimensional spaces. This paper shows that the well understood theory of sampling, as studied in communications theory, can carry over into the two dimensional and higher dimensional cases.

2.3.2 Differentiation.

Differentiation is primarily a technique for enhancing the abrupt changes in the image, i.e. at lines and edges, and places where the grey scale changes. Techniques used for this include the classical gradient operator (ROB65) and Laplacian edge detection operators. A generalisation of the digital gradient in conjunction with a scheme for automatic threshold selection has been developed (CHO72). A generalised Laplacian used in conjunction with counts of the values above and below that of the point in question, to distinguish straight, convex and concave edge points, has been reported. The motivation for spatial differentiation is that natural scenes usually consist of objects with sharply defined surfaces. Therefore, two dimensional projections of natural scenes often consist of domains of different brightnesses separated by well defined edges. It follows that the edge is a fundamental primitive in images, and differentiation enhances edges and lines.

One of the problems of differentiation operators is that they are extremely sensitive to noise in the image; operators which both average and difference have been developed using windows of various

sizes (DUD73). Logical operators have also been proposed which look at the area around the point of interest in the image (MAR72,KEL71). These operators are normally faster than differencing operators and they tend to reject false edges, although they sometimes miss true edges.

2.3.3 Averaging.

Averaging, or smoothing as it is often known, is a technique for cleaning noise from an image, or at least minimising the effect of noise on the image, by blurring or de-emphasising sharp changes. A number of recent techniques developed include a smoothing technique which uses only neighbouring points that pass a simple statistical test and a dynamic programming approach which works by minimising both the degree of irregularity of the smoothed image and its difference from the noisy image. As described for differentiation techniques, logical averaging or logical smoothing has also been developed (DOY60). It is based on the observation that the points appearing within the averaging window can be treated as Boolean or logical variables, and the value of the smoothed image point can be defined by any Boolean function on these variables.

The basic difficulty with noise removal and smoothing techniques is that, if applied indiscriminately, they tend to blur the image, and this is usually unacceptable. In particular, one usually wants to avoid blurring sharp edges or lines that occur in the image.

2.3.4 Template Matching.

Template matching is a technique for segmenting an image into preselected parts by cross-correlating a known pattern with the image

and identifying points of high correlation. In a great variety of image analysis problems the analyser is confronted with one variation or another of a single simple question: 'Does the image contain within it a previously specified object?'. We can use basically two types of templates; local templates and global templates. (References MUN68 and HIG61 give respectively a description of local and global templates in character recognition.) A little reflection will lead us to the conclusion that the problem of template matching can only be solved by having a whole family of templates. The difficulty is that the object we are looking for can be viewed at any angle or rotation. In such situations a common approach is to replace the global template with a set of local templates. The local templates are designed to match various parts of the object of interest, the rationale being that the individual parts vary less in appearance than an entire object (ROB65).

2.3.5 Region Analysis.

Region analysis attempts to simplify a digital image by partitioning it into a set of disjoint regions. In the simplest case, each region is composed of points having the same grey value and connected to each other. The precise notion of when two points are connected is given, by defining whether a cell is connected to all eight surrounding cells (8-connected) or whether it is connected only to those four cells sharing a common edge (4-connected). This is defined precisely in mathematical terms by J.P. Mylopoulos and T. Pavlidis (MYL71) who also discuss the effect on different algorithms of eight or four connectedness.

The first major paper on region analysis, by C.R. Brice and C.L. Fennema (BRI70) demonstrated how effective region analysis

techniques could be if a method was developed for merging spurious regions. A method for growing and merging regions was also used in the Edinburgh University robot project (BAR70). Clustering techniques have also been used to decompose images into regions (HAR69). A variation on the region analysis method is to decompose an image into a tree structure of constituent parts (KIR71). This decomposition is viewed as 'the morphological precursor to a higher level syntactic analysis'. The method is not only concerned with recognising a pattern but also the naming of its structural parts, with an indication of their relations to each other.

2.3.6 Contour Following.

Contour following involves tracing out the boundary between a figure and its background (MER73). If we imagine a contour follower as a 'bug' which crawls across an image until it enters a 'dark' region, i.e. the figure, it then curves to the left until it leaves the figure, at which point it immediately begins to curve to the right. Repeating this process, the bug will follow the figure boundary in a clockwise direction until it returns to the neighbourhood of the starting point. The set of transition points between the figure and background can be taken to be the contour.

There are two major conditions that must be fulfilled by a contour following algorithm if it is to be successful. First, since an image generally has many grey levels, there must be some way of defining the figure whose boundary is to be followed. In simple cases the figure can be extracted from the background by the simple expedient of thresholding the image. We merely state that everything in the image darker or lighter than a fixed value is figure. When this cannot be done, it is sometimes possible to follow a contour

based on a sufficiently large change in grey values between the figure and background. Some algorithms accomplish this by combining a gradient operator with a contour follower (ROS66). The gradient operator finds the local direction of the contour (assuming a point on the contour has been found to begin with) by finding the direction of the most prominent edge near a contour point. A small step taken in this direction is assumed to result in a new point near the figure boundary. The gradient operator is called to find a prominent edge near the point, and the process is repeated.

The second major prerequisite for successful contour following is that the figure has no spurious gaps in it. This difficulty can sometimes be overcome by first 'smoothing' the image in order to fill in small gaps (ZAH69). H.D. Kelly (KEL71) has reported a novel 'planning contour follower' that first obtains a rough approximation to the contour from a reduced resolution version of the image.

2.3.7 Enhancement.

Image enhancement, or 'image filtering' as it is often known, is a method for compensating for the effects of a specific (known or estimated), degradation process. Successful use of filtering theory can be made if the actual degradation process that has operated on the image can be estimated (JAI74). The reason that linear filtering techniques are not applied successfully to images is that the noise or degradation of the image is very seldom linear. More elementary methods of filtering or enhancing images have been tried. These include methods of modifying the grey scale, e.g. increasing contrast, deblurring, smoothing or removing identifiable noise points, and correcting geometrical distortions (JOH70, OHA72). In all but the last of these, little or no attempt is made to estimate the

actual degradation process. These methods, however, do take into account certain general properties of image degradation.

2.3.8 Transformations.

The transformation of images into other domains has been used for a number of different purposes. Some work has been done on the direct analysis of the image's transform (HOL65,LEN70). An approach to image compression that makes use of invertible transforms has received considerable attention during the past few years (AND71,AND72). The basic idea is to encode or approximate, not the image itself, but its transform, and then to reconstruct the image by inverting this transform. Under certain circumstances, this may permit a greater degree of compression without loss of acceptability or may provide greater immunity to noise. The Fourier transform was the first studied in this connection, but a wide variety of others, particularly the Hadamard transform, have also been investigated (PRA69). Another use of transforms has been in the area of image restoration and spatial filtering. However, these processes require information not only about the degradation operation but also about the undegraded object, and in practice this information can only be roughly approximated. Classically this approach has used Fourier transform methods (BRA68,LIN66). An important reference work for these transform techniques in image analysis is a text book by H.C. Andrews (AND70).

2.3.9 Line Fitting.

Line fitting is a method for joining a sequence of points, which have been identified as lying on edges or lines, into identifiable lines or curves. An early system fitted points, (which

were identified as points of large gradient) into lines using a local template operation (ROB65). Several investigators have studied mathematical methods for defining edge detectors that are optimum in various senses. One method finds the perfect step edge that best matches the given digital image in a certain disk-shaped neighbourhood of each point (HEU71,HEU73).

Curve detection is most commonly done by tracking edge points. Elaborate detection operations are then only necessary at points where obvious curves are suspected by simpler operations. This method also permits the detection criteria to vary from point to point with the nature of the curve being tracked (MON71). A procedure has also been reported which detects circles and approximately circular arcs of varying grey levels in an edge enhanced image (KIM75). This procedure is an extension and improvement of the circle finding concept sketched by R.O. Duda and P.E. Hart (DUD72) as an extension of the Hough straight line finder (ROS69).

2.3.10 Line Description.

Line description is a method of describing the objects in an image by defining the boundary lines of the object. The general approach is to fit the figure by one or more straight lines or perhaps polynomial curves. There are two problems: where to break the original line into segments, and how to fit a line to each segment. A number of techniques have been proposed to solve these problems: minimum squared error line fitting (WIL62), eigenvector line fitting (WIL62), line segmentation by iterative end point fits (DUD73), and breaking lines at points of maximum curvature (ZAH69). Another commonly used method for describing lines is chain encoding (FRE61a). This method uses the start point of the line followed by a list of

the direction changes of the curve to describe the line. Chain encoding is particularly useful for comparing the shape of two curves, as it can be done by chain cross-correlation (FRE61b).

2.3.11 Shape Description.

The shape descriptions which are most interesting are sufficiently general to be invariant under certain sets of transformations. The challenge here is to find descriptions that are invariant to transformations leaving the figure unaltered in 'unimportant' ways, yet sensitive to transformations that change the figure in 'important' ways. The use of invariant properties dates back to the earliest days of image analysis. Geometrical normalisation techniques have been used in various applications (FIS69,MUN68). Distance functions on digital images were studied by A. Rosenfeld and J.L. Pfaltz (ROS68). Finding the medial axes or skeleton of a shape has also been used most notably in chromosome analysis (HIL69).

2.4 THERMOGRAPHY.

Thermography has been used as a tool in the early diagnosis of breast cancer. Scanning thermometers have been developed which can scan the area of diagnostic interest in less than a second. This scanning is free from any medical dangers as it relies entirely on the fact that the human body radiates energy as heat. This radiation is sampled and represents the absolute surface temperature of the body. As they are relatively cheap, harmless and easy to take, thermographic scans have been proposed as a screening test for breast cancer. The main disadvantage is that the data collected is

inherently quantitative and therefore expensive in time and effort for human interpretation. Some of the literature on various aspects of thermography is covered below.

2.4.1 Thermographic scanners.

A thermographic scanner is an infra-red sensitive camera which measures the black body temperature in its field of view. Calibration stability is achieved with the aid of built-in thermal references (OSB71). Thermographic investigations in medicine have indicated the need to resolve temperature in the range 28 degree centigrade to 37 degree centigrade over an area of up to 40 centimeters by 30 centimeters. The spatial resolution requirement is 3 millimeters, and a temperature measurement accuracy of .02 degree centigrade is thought desirable (DOD68,LAW63). The Atomic Weapons Research Establishment MkII scanner, which was used to collect the data used in this thesis, was constructed to facilitate quantitative medical thermography (OSB71). A primary objective of this scanner was to produce accurate information in a numerically evaluated form. Stable thermal references were included in the scanner to eliminate calibration problems and an unusually efficient optical system allows good spatial resolution and signal to noise characteristics, without sacrificing speed. The detector is a liquid nitrogen cooled photoconductive element of indium antimonide.

2.4.2 Breast cancer diagnosis using thermography.

Research has shown that absolute temperature has no diagnostic value (LAW63). A simple algorithm has been used for diagnosis at the Royal Infirmary of Edinburgh. It tries to measure temperature asymmetry of the breasts and pick out the presence of a localised

'hot spot' (SC076). The thermal resolution of a thermographic scanner required for use in breast cancer detection has been derived from work by R.W. Lawson and M.S. Chugatai (LAW63); this has shown that the average temperature difference between a malignant tumour and the contralateral site is 1.5 degree centigrade.

2.4.3 Breast cancer screening using thermography.

Screening for breast cancer using thermography is not currently available in the United Kingdom under the National Health Service. A breast screening programme has been set up in Edinburgh and two other cities in Britain to test the feasibility of such breast screening clinics and to act as a pilot scheme for the possible development of clinics under the National Health Service. A program of research called the H.I.P. study was carried out in New York this suggested that approximately a 14% increase in the five year survival rate of breast cancer sufferers could be achieved through screening (LIL69). However, doubt has been cast on the validity of these figures by various authors.

2.4.4 Breast cancer statistics.

Breast cancer is the most common cause of female cancer deaths. Between 11000 and 12000 women die every year in the United Kingdom alone i.e. about one in five of all cancer deaths. Breast cancer is three times more common than any other female cancer; in addition it is one hundred times more common in females than in males (REG67).

2.5 VISUAL PERCEPTION.

Knowledge about the human visual system can be very useful to the designer and user of image analysis techniques. In particular, one must know something about subjective image quality and about the fidelity of an image to the original scene, when designing systems for image digitisation and coding or for image enhancement, when the image is intended for viewing by humans. Similarly, when analysing the structure of an image for purposes of image description, one wants to extract image parts that correspond to those seen by humans, and to describe them in terms corresponding to those used by humans.

An important point to keep in mind when considering the human visual system is that it cannot be treated simply as a special type of image digitisation and transmission system. To regard it as such is to commit the 'homunculus fallacy'. (When an image has been transmitted by the eye to the brain, there is no 'little man' inside the brain to look at it!). The input to the visual system may be an image, but the output which it furnishes to the higher brain centres must be something quite different. This fact has many implications as regards the perception of even the simplest stimuli.

An example of this conceptual difficulty has been demonstrated for edge finding. Edge finding may seem easy to the reader because the human visual system extracts useful edge information in the presence of a variety of disturbances. However, it seems from the work of Hubel and Wiesel (HUB63) that the retina may do the work of edge abstraction before the visual information is passed to the brain. Another difficulty encountered in attempts to automate aspects of the human visual system is that it may be a function for which the

human visual system is poorly adapted. Two examples are quantitative brightness estimation and noncontiguous area comparison (ZUS70,COR70), both of which are important in breast thermogram interpretation. Another paper (HUB62) summarising the work of Hubel and Wiesel shows that cats evidently do the equivalent of local template matching when detecting the presence of straight line segments.

3. IMAGE ANALYSIS TECHNIQUES.

3.1 Introduction.

3.2 Digitisation.

3.3 Differentiation.

3.4 Averaging.

3.5 Template Matching.

3.6 Region Analysis.

3.7 Contour Following.

3.8 Enhancement.

3.9 Transformations.

3.10 Line Fitting.

3.11 Object Description.

3.12 Line Description.

3.13 Shape Description.

3.1 INTRODUCTION.

This chapter classifies similar image analysis techniques and explains the principles on which each of the different methods is based. Not all of these techniques would be required for any single image analysis application but they represent the range of techniques which are necessary for a general purpose system. All of them are essentially concerned with the computer recognition and

classification of meaningful regularities in images. These techniques are not explored in detail in this introduction, as they are dealt with later in the chapter, but I would like first to clarify in mathematical terms exactly what is meant by an image and an image property.

One of the first problems is to select a representation of the image that is suitable for the machine. There are many means of doing this, but they can be put into perspective by recognising that a black and white image can be thought of as a real valued function of two variables. To be specific, suppose that the image occupies a plane with x, y cartesian coordinates. An image function $F(x,y)$ is defined to be proportional to the light intensity impinging on (or the heat radiating from) the image at the point (x,y) . The intensity of an image at a point is also called the grey level or the brightness. In any event, from a mathematical point of view an image is defined by specifying its image function.

Once we have agreed to consider images as functions, it is obvious that any means of representing a function in a digital computer can be used to represent an image. Some methods can be dismissed immediately as being inappropriate. Polynomials, for example, are generally represented by storing their coefficients, but few interesting image functions are merely low degree polynomials. More generally, an interesting image function seldom has a simple analytic form, and so its representation is usually accomplished by sampling the image function at a discrete number of points in the x - y plane and storing sample values. This process is called 'sampling' or 'quantisation'. In order to define a quantisation algorithm, we must first specify where the samples are to be taken. The simplest approach is to partition the image plane by a quadruled grid and to

sample the image function at the centre of each cell. Now in general the image function can assume any value between some minimum (black) and some maximum (white) or vice versa, while the digital computer can represent only a finite number of values. In addition, therefore, we must also partition the range of amplitudes of the image function into a finite number of levels. If this is also done, we shall have specified an algorithm for representing the original image function as an array of integers, where each element of the array specifies the approximate grey level of the image in the corresponding cell.

For the remainder of this thesis then, an assumption will be made wherever convenient that an image can be represented by an ordinary matrix whose values define the approximate values of an image function in the corresponding region of the image plane. When there is no danger of confusion, call the (x,y) element of the matrix $F(x,y)$ and in this case speak of F as a 'digital image function'. The unquantised image function $F(x,y)$ will be referred to as the 'analogue image function'.

An 'image property' is a function that maps an image into a number; the number obtained from a given image F is the value of the property for that image. Examples of properties are:

- 1) The grey level of $F(x,y)$ at some specified point.
- 2) The average grey level of F , i.e. $(1/A) \iint F(x,y) dx dy$
where A is the area of the region of integration.
- 3) A particular Fourier coefficient of F say
 $\iint F(x,y) \cos(mx+ny) dx dy$.

Such properties can be computed directly as soon as F is given. Other types of property, on the other hand, can be computed only after some preliminary operation has been performed on F . Geometrical properties of image parts, for example, can only be measured after the parts in

question have been, at least implicitly, extracted from the image by segmenting it. Given an image subset S , examples of properties that can then be measured are:

- 1) The number of connected components of S .
- 2) The areas of these components.
- 3) The position of the centroids of these components.

In some cases, the segmentation and property measurement can be performed in a single operation. For example, when we match a template G against an image F , say by cross-correlating them, we can take the highest value of the cross-correlation as a property which measures the degree of match between G and F . Many other transformations of an image, besides segmentation, can be useful as preprocessing operations to facilitate property measurement. For example, one can sometimes obtain 'cleaner' property values by enhancing the image, e.g. by cleaning up noise before measuring the properties. Many image properties are most conveniently measured if we first compute some transform of the given image, e.g. its power spectrum. Other properties are conveniently measured by first computing some function of a single variable from the given image or image subset, e.g. a projection or cross section, an intrinsic equation, or a histogram. One often wants to define properties whose values are not affected by changes in the brightness, contrast, position, orientation, size, etc. of the given image or image subset.

Most of the image properties that are used for image classification and description are chosen on heuristic grounds, i.e. because they are easy to measure and appear to be relevant to the desired goals. In most cases, there is no mathematical theory that can be used to determine optimal properties of given types for given purposes. Another factor becomes important if we want not merely to

classify images but to describe them in the same way as people do. The properties used in such descriptions must be compatible with those used by the human visual system; otherwise meaningful dialogue about the images between humans and machines would be difficult or impossible.

An image property is called 'local' if its value depends only on some small portion of the given image. For example, the value of a local operation at any point such as differentiation is a local property, since it depends only on the part of the image in the immediate neighbourhood of that point. The extreme case of a local property is a single point property whose value depends only on a single point in the image. Statistics of local property values provide useful information about the 'texture' of an image. For example, in a 'busy' image the average value of the gradient or of the Laplacian should be high, whereas in a smooth image it should be low. In a highly directional image, say one containing many streaks in a direction given by an angle θ , the average value of the directional difference in the direction given by θ should be low, while that in the direction given by $\theta + 90$ degrees should be high.

In attempting to analyse an image, one should certainly make use of whatever prior knowledge is available about the class of images to which the given image belongs. Such knowledge can be regarded as a model, perhaps a very informal one, for the class of images in question. The model should be used to guide the choice of image processing, segmentation, and property measurement operations that will be performed on the given image.

3.2 DIGITISATION.

As defined above, images can generally be considered continuous functions, the grey level being a continuous function of position in the image. Before such an image can be processed by a digital computer, or in many cases, transmitted over a channel, it needs to be digitised.

In its ordinary sense digitisation consists of sampling the grey level in the image at an M by N array of points. Since the grey levels at these points may take any values in a continuous range, for digital processing the grey level needs to be quantised. This means that the range of grey level is divided into K intervals, and it is required that the grey level at any point take on only one of these values. In order for the image reproduced from these numbers to be a 'good' representation of the original, M , N and K have to be large. Ordinarily, the finer the sampling and quantisation, the better the reproduced image. Nothing is gained, however, by increasing M , N and K beyond the spatial and grey scale resolution capabilities of the receiver.

In a more general sense, the aim of sampling is to represent a continuous image by a finite array of numbers, called samples. The only constraint on these numbers is that it should be possible to reconstruct an image from them. There may be small errors involved in the reconstruction, provided that these errors do not cause any impairment in the representation of the scene with respect to the spatial and grey scale resolution capabilities of the receiver.

There are two different approaches to image sampling. In the first approach, the samples are the values of the image function (grey levels) at a discrete array of points. There are constraints which an image must satisfy for this type of sampling to be adequate.

In the second approach, the image function is expanded in terms of orthonormal functions, and the coefficients of expansion are the samples. However, in digital processing the image samples must be quantised. This means that the range of values of the samples must be divided into intervals and all the values within an interval represented by a single level.

The specific question of how finely an image must be quantised in order to preserve all of its information is now dealt with. The primary theoretical result bearing on the question is the celebrated Shannon sampling theorem (SHA49).

The basic idea behind the sampling theorem rests on the correspondence between abrupt changes in the intensity of an image and high spatial frequencies in its transform. If an image's transform contained no high frequencies then the image itself could not contain abrupt grey level transitions, so we might suspect that such an image need not be quantised very finely. Accordingly, we call an image function $F(x,y)$ 'bandlimited' if its Fourier transform $FT(f_x,f_y)$ is zero whenever either $|f_x|$ or $|f_y|$ is greater than some number W . The Shannon sampling theorem states that a bandlimited function can be reconstructed exactly from image samples taken a nonzero distance apart and specifies how this reconstruction is done.

The greater the bandwidth W , the smaller the sampling interval $1/2W$ must be. This is merely another way of saying that a larger bandwidth means that the image function can 'change' faster, so it must be sampled more finely in order to capture all the 'changes'. A two dimensional image function can be completely defined by samples of it taken on a quadrupled grid with spacing $1/2W$. A minor complication is introduced by the fact that an image function can

have different bandwidths in the f_x and f_y directions, such that

$$FT(f_x, f_y) = 0 \text{ whenever } |f_x| > W_x \text{ or } |f_y| > W_y.$$

If this happens samples can be taken on a rectangular grid rather than a square one, with spacing $1/2W_x$ and $1/2W_y$ in the x and y directions respectively.

3.3 DIFFERENTIATION.

As mentioned earlier, the process of image analysis is generally one of simplification, i.e. the original image of a complicated object is converted into a simpler form by some sequence of steps. In such a sequence, one natural step is to convert the given image into a line drawing. We would hope that this step would preserve the important features of the original image but would reduce the computational requirements imposed on subsequent steps. There are also psychological grounds for considering outline drawings of the image. Experiments have shown that humans concentrate most of their attention on the borders between more or less homogeneous regions. Of course, we recognise that in reducing an image to a line drawing we face certain hazards. The reduced image generally contains less information than the original and there is no guarantee that the information lost is irrelevant. However, assuming that the reduction of an image to an outline drawing is useful in some circumstances, let us now turn our attention to means of accomplishing this.

An outline drawing of an image can be produced by emphasising regions containing abrupt light-dark transitions and de-emphasising regions of homogeneous intensity. In other words, outlines are edges, and edges are by definition transitions between two markedly different intensities. In terms of the image function, an edge is a

region of the x-y plane where $F(x,y)$ has a gradient with a relatively large absolute value. The problem of producing an outline drawing thus requires estimating the magnitude of the gradient of a function. Now the gradient can be estimated if we know the directional derivatives of the function along any two orthogonal directions. The process of obtaining the gradient image is variously known as spatial differentiation, edge enhancement, sharpening, or simply taking the gradient. When an image is noisy as well as blurred, differentiation cannot be used indiscriminately to sharpen it, since the noise generally involves high rates of change of the grey level, it usually becomes stronger than the image signal at high frequencies. These methods should be restricted, if possible, to frequency ranges where the image is stronger than the noise.

Turning now to specific mathematical techniques, a partial derivative operator $D = (dn/dx_k)dy(n-k)$ is a linear operator. It follows therefore that any linear combination of D s is also a linear operator. An interesting class of derivative operators are those which are isotropic, i.e. rotation invariant. We want our operators to be isotropic because we want to sharpen blurred features, such as edges and lines, that run in any direction. It can be shown that:

- 1) An isotropic linear derivative operator can only involve derivatives of even orders.
- 2) In an arbitrary isotropic derivative operator, derivatives of odd orders can only be raised to even powers.

The Laplacian is the linear derivative operator

$$D^{**2}F = d^{**2}F/dx^{**2} + d^{**2}F/dy^{**2}$$

It is rotation invariant. Modifications to this method have been proposed, with the aim of reducing noise sensitivity. One suggestion is to use, instead of the Laplacian, the second partial derivative in

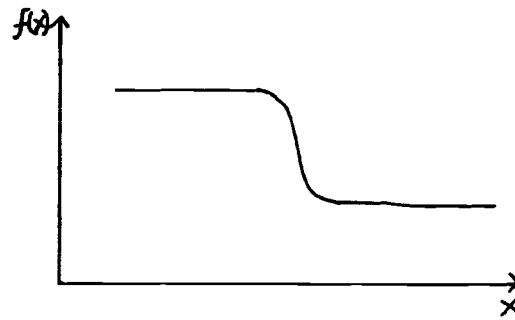
the gradient direction ∇n . A further refinement is to use a linear combination of the second order partial derivatives in the two perpendicular directions ∇n and ∇t . This last scheme can be designed to smooth the image in the direction along an edge at the same time as it sharpens the edge. For a digital image, the discrete analogue of the Laplacian is

$$\begin{aligned} \Delta^2 F(i,j) &= D_x^2 F(i,j) + D_y^2 F(i,j) \\ &= (F(i+1,j) + F(i-1,j) + F(i,j+1) + F(i,j-1)) - 4F(i,j) \end{aligned}$$

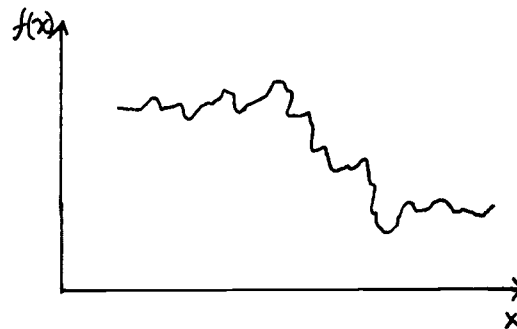
Note that this is proportional by the factor $-1/5$, to

$$F(i,j) - 1/5 (F(i+1,j) + F(i-1,j) + F(i,j) + F(i,j+1) + F(i,j-1))$$

which is the difference between the grey level $F(i,j)$ and the average grey level in a neighbourhood of (i,j) , where the neighbourhood consists of (i,j) and its four horizontal and vertical neighbours. Thus we see that the digital Laplacian of an image F is obtained, up to a constant factor, by subtracting a blurred, i.e. averaged, version of F from itself. Figure 2 shows how an averaging operator should work.



An Idealised Edge.



An Actual Edge.

Notation

A	B	C
D	E	F
G	H	I

A smoothing operator

$$S_x = (C+2F+I)-(A+2D+G)$$

$$S_y = (G+2H+I)-(A+2B+C)$$

$$\text{at point E } S = \sqrt{S_x^2 + S_y^2}$$

Figure 2. Sobel's Operator.

Derivative operators, which give high values at points where the grey level of the image is changing rapidly, were discussed above. Any such operator can be used as an edge detector; its value at a point represents the 'edge strength' at that point, and we can explicitly extract sets of edge points from the image by thresholding these values.

The simplest derivative operators are the first partial derivatives dF/dx and dF/dy , which give the rates of change of the grey level in the x and y directions. The rate of change in any direction θ is a linear combination of these and is

$$(dF/dx)\cos\theta + (dF/dy)\sin\theta.$$

The direction θ_n at a given point in which the partial derivative has greatest magnitude is

$$\tan^{-1}((dF/dy)/(dF/dx)), \text{ and the magnitude is } \sqrt{(dF/dx)^2 + (dF/dy)^2}.$$

The vector having this magnitude and direction is called the gradient of F . For digital images, we can use differences in place of derivatives. Thus the magnitude of the digital gradient of F at (i,j) is

$$\sqrt{D_x^2 F(i,j) + D_y^2 F(i,j)}.$$

It is common practice to approximate this expression, either by

$$|D_x F(i,j)| + |D_y F(i,j)| \text{ or by } \max(|D_x F(i,j)|, |D_y F(i,j)|).$$

Note, however, that these approximations are no longer equally sensitive to edges in all directions. They agree with the exact expression only for horizontal and vertical edges. Some other popular digital gradients are

$$\max(|F(i,j) - F(i+1,j+1)|, |F(i+1,j) - F(i,j+1)|) \text{ Roberts.}$$

$$F(i+1,j) + F(i-1,j) + F(i,j+1) + F(i,j-1) - 4F(i,j) \text{ Laplace.}$$

Finding an edge in a defined neighbourhood can be thought of as a minimisation problem. A solution to this problem was derived by M.H. Hueckel (HUE71, HUE73) for the case of a circular neighbourhood D of unit radius, using a truncated series expansion in terms of a set of basis functions defined on it. This set of functional equations expresses the requirement that the functions have decreasing weight as one moves away from the centre of D and that they be insensitive to high frequency noise. The reader is referred to Hueckel's papers for the details, which are somewhat involved.

3.4 AVERAGING.

In this section some simple methods of noise removal, or more generally, of making an image 'smoother' are described. The basic difficulty with noise removal and smoothing techniques is that, if applied indiscriminately, they tend to blur the image, which is usually undesirable. In particular, we usually want to avoid blurring sharp edges or lines that occur in the image.

If the noise occurs in known positions in the image, or we are able to distinguish the noise from the rest of the image, it becomes relatively easy to remove the noise without bad effects on other parts of the image, since we can operate only on the noise and leave the rest of the image intact. As a simple example, suppose that a periodic line pattern V has been added to the image F . We can remove the lines, without affecting the rest of the image, by changing the grey level of each line point to the average of the levels of the neighbouring image points. Alternatively, we can remove the lines by filtering in the Fourier domain. If we take the Fourier transform of the noisy image $F+V$, we obtain the sum $f+n$ of the transforms of F and V . Now the Fourier transform n of the line pattern V has all of its 'energy' concentrated at a set of small spots along a line perpendicular to the direction of the lines. If we suppress these spots from the transform (zero them or remove them by interpolation) and then take the inverse transform, we obtain a smoothed image in which V has been deleted, while F is relatively unaffected.

It should be pointed out that interpolation to replace the removed noise is easy when the noise comprises fine isolated points, thin lines etc., so that each noise point has non-noise neighbouring



points. When the noise is coarse (large artefacts, wide bars, etc.) it becomes much harder to replace it inconspicuously.

The smoothing methods described in the preceding paragraphs depend on first distinguishing noise from non-noise in the image, then removing the noise and 'mending' the image by interpolation. Turning now to another class of smoothing methods which do not depend on identification and removal of noise. Instead, these methods weaken the noise by applying some type of averaging to the image. Since averaging blurs the image, these methods must be applied with care, to avoid degrading sharp details. We can perform the averaging without any danger of blurring when we are given several independently noisy copies $F_i(x,y)$ of an image, where the noise values of the copies at a given point are independent. In this case we can reduce the noise by pointwise averaging the copies, thus obtaining the new image F defined by $F(x,y) = (1/n) \sum_i F_i(x,y)$.

If we only have a single noisy image available, we can attempt to reduce the noise level by local averaging, i.e. by giving each point a new grey level which is the average of the original grey levels in some neighbourhood of the point (including the point itself). This is relatively straightforward if the noise is finer grained than the smallest image detail of interest. It should then be possible, by averaging over a small neighbourhood of every point, to reduce the noise while keeping the blur at a negligible or at least tolerable level.

We call this process 'regularising' the function. We define the regularised function $F_w(x,y)$ by:

$$F_w(i,j) = (1/((2b+1)(2h+1))) \sum_{-b \leq m \leq b} \sum_{-h \leq n \leq h} F(i+m, j+n)$$

where $w(x,y)$ is a window of area $A_w = (2b+1)*(2h+1)$ centred on cell (i,j) . Qualitatively, regularising an image has very much the same

effect as defocusing a camera, with a large averaging window corresponding to a camera severely out of focus. This is often precisely the opposite of the required effect. We prefer as sharp and clear an image as can possibly be obtained. Consequently regularisation must be used selectively, for special purposes.

If we are dealing with binary images, there is an alternative smoothing technique that often allows us to exercise more precise control than does regularisation. This technique is called logical averaging or logical smoothing. It is based on the observation that the image elements appearing within the averaging window can be treated as Boolean or logical variables, and the value of the smoothed image function at a point can be defined by a Boolean function of these variables. Boolean operators can be constructed to combat special noise idiosyncracies or to take advantage of prior knowledge about the figures likely to be encountered.

3.5 TEMPLATE MATCHING.

In a great variety of image analysis problems the analyser is confronted with one variation or another of a single simple question: 'Does the image contain within it a previously specified object?' The technique classically applied to answer this question is called template matching.

A very simple approach to the problem might involve making a template or mask, and scanning it systematically across the entire image. We accept that an object has been found only if each region of the template covers an area of the image whose grey values correspond to those of the template. In other words, the 0 regions of the template must 'show' only grey values of 0, and the 1 regions must

show only grey values of 1. The size of the template is typically smaller than the size of the original image, since our aim is to discover the presence of a 'sub-image' within the given image. Mathematically, we say that the domain of definition of the template is smaller than the domain of definition of the original image.

In most practical applications, one would not expect to find the perfect template match contemplated above. A more realistic approach would be to define some measure of how well a portion of an image matched a template. One such definition is the following:

Let $F(i,j)$ be our digital image,

Let $T(i,j)$ be our template, and

Let D be the domain of definition of the template.

Then a measure of how well a portion of the image matches the template can be defined as:

$$M(m,n) = \sum_i \sum_j (F(i,j) - T(i-m,j-n))$$

such that $(i-m,j-n)$ is in D .

A little reflection will lead us to the conclusion that the problem as stated can only be solved by having a large family of templates. In the absence of any explicit constraints, our object can appear anywhere in the image and can be viewed from any angle. For each orientation we evidently need a separate template and since each template must be scanned across the image, the computational burden is likely to be large. In such situations a common approach is to replace the 'global' template by a set of 'local' templates. The local templates are designed to match various parts of the object of interest, the rationale being that individual parts vary less in appearance than the entire object.

This approach is reasonable in a large class of problems whose hallmark is a wide variability in the appearance of the object of

interest. On the other hand, there are some pattern matching problems that can be sufficiently constrained or stylised to ensure that each pattern will always appear in isolation and at the same size and orientation. The choice of whether to use local or global templates is primarily dictated by the anticipated variability in the images to be processed.

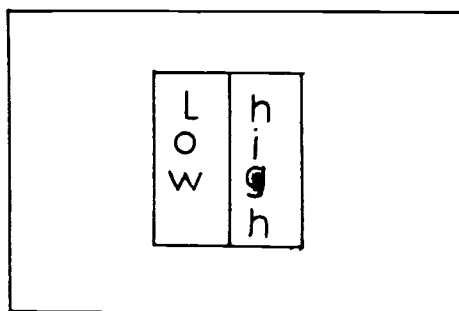


Figure 3. A Local Template. ;
A template for vertical edges.

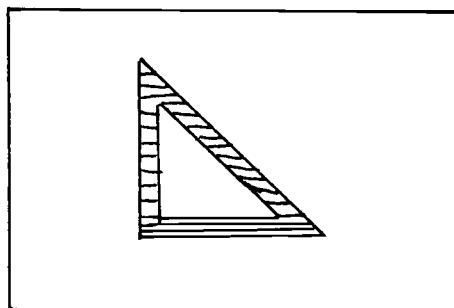


Figure 4. A Global Template.
A simple template for a triangle.

As well as the method described above, there are many possible ways of measuring the degree of match or mismatch between two functions F and T over a region R . For example, one can use as mismatch measures such expressions as $\text{MAX}(R)!F-T!$ or $\iint R!F-T!$ or $\iint R(F-T)**2$ and so on (where integrals become sums in the digital

case). It is easily verified that these expressions are all 'distance measures' or metrics.

For digital images, cross-correlations would normally be computed point by point; for each relative shift (u,v) of F and T , we multiply them pointwise and sum the results to obtain $CFT(u,v)$. If F is m by m and T is n by n , where n is usually much smaller than m , this requires n^2 multiplications to be performed for each of m^2 relative shifts. By the convolution theorem, cross-correlating a template T with an image F is equivalent to pointwise multiplying the Fourier transforms F^* and T^* and then taking the inverse transform. This method can be faster than direct cross-correlation, if an efficient Fourier transform algorithm is used. Note that the Fourier transform arrays that we multiply pointwise should be of the same size. To achieve this when the template is smaller than the image, we can fill out the template to the size of the image by adding rows and columns of zeros, or some other constant value.

It should be noted that an important aspect of template matching, whether performed on the original image or on a processed version of the image, is that the operation makes use of only local information. If we are trying to decide whether a vertical line is present in a given region of the image, the only thing that affects our decision is the set of intensity values in that region. This local aspect is both the reason for the appealing simplicity of template matching and the source of its most stringent limitations.

3.6 REGION ANALYSIS.

Region analysis attempts to simplify a digital image by partitioning it into a set of disjoint regions. In the simplest case,

each region is composed of cells having the same grey value and connected to each other. There are two definitions of when two cells are connected, they depend on whether a cell is connected to all eight surrounding cells (8 connected) or whether it is connected only to those four cells sharing a common edge (4 connected). See figure 5 for a visual explanation. It should be noted in passing that a hexagonal grid does not suffer from the problem of ambiguity which 4 and 8 connectedness have, since any two hexagonal cells with a common vertex also share a common side.

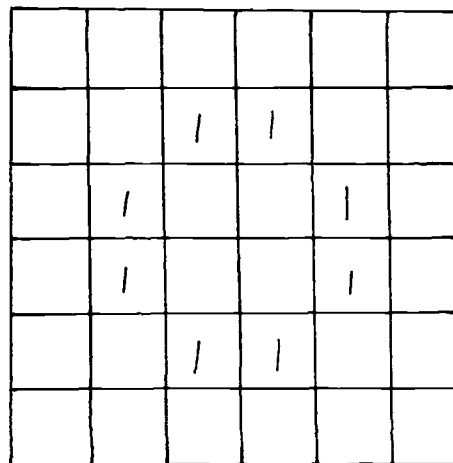


Figure 5. 4 and 8 Connectedness.

4-Connected: cells sharing a common side are connected.

- 1) The figure is not connected.
- 2) The background is not connected to the hole.

8-Connected: cells sharing a common vertex are connected.

- 1) The figure is connected.
- 2) The background and hole in the figure are connected.

With the given definition of connectedness, the question of analysing an image by partitioning it into regions can be examined. A set of cells R is an elementary connected region if:

- 1) All the cells in R have the same grey value.
- 2) Any two cells in R are connected by a chain of adjacent cells each of which is in R.
- 3) Any set of cells that properly contains R fails to satisfy these conditions.

The second condition can be taken as the definition of a connected region. The third condition ensures that the regions are as large as possible.

A more mathematical definition of the problem is as follows let $F(x,y)$ be an image function of an image defined on a domain R (usually consisting of M by N pixels). It is required to divide R into a minimum number of regions $R_1, R_2, R_3, \dots, R_n$ where $F(x,y)$ satisfies certain constraints, e.g. is approximately constant.

Unfortunately, using the above definition many small regions are usually found. There are at least two ways of combatting this: by relaxing the first condition, or by merging regions. With the first alternative, two adjacent cells are put into the same elementary region if the difference in grey levels is less than a specified threshold. Alternatively, the elementary connected regions, as originally defined, are formed and some criterion is used to decide when to merge two regions sharing a common boundary.

Another simpler, and probably commonest way to extract regions from an image is to threshold the image. If the given image F has a grey level range Z_1, \dots, Z_k and T is any number between Z_1 and Z_k , the result of thresholding F at T is the two valued image function F_T , defined by

$$F_T(x,y) = 1 \text{ if } F(x,y) \geq T, = 0 \text{ if } F(x,y) < T.$$

We can also consider thresholding operations that map other specified ranges of grey levels into 1, and levels outside these ranges into 0.

e.g. $F_u(x,y) = 1$ if $F(x,y) \leq u$, 0 if $F(x,y) > u$

$F_{uv}(x,y) = 1$ if $u \leq F(x,y) \leq v$, 0 otherwise.

More generally, if Z is any set of grey levels, $Z = (Z_1, \dots, Z_k)$ we can define a generalised 'thresholding operation' as mapping the grey levels in Z into 1 and those not in Z into 0.

i.e. $F_Z(x,y) = 1$ if $F(x,y)$ is a member of Z , 0 otherwise.

A useful variation on thresholding is what we might call 'semi-thresholding'. Here certain grey levels, e.g. those below a threshold, become 0, but the remaining grey levels are unchanged, rather than becoming 1. Applied to an image that consists of objects on a background, this technique can be used to 'zero-out' the background while preserving the grey levels within the objects.

Other methods include one in which it is useful to threshold a processed image F' , rather than the original image F . Here the only grey levels in the image are black and white, but the probability of black occurring in the object is higher than in the background, so that the object has a darker average grey level than the background. Thus if local averaging is applied to F , a processed image F' is obtained in which the object now consists of darker grey levels than the background, so that it can be extracted by simple thresholding. In the example just given, an object that had a different average grey level from its background was extracted by locally averaging and then thresholding. More generally, suppose that the object differs from its background with respect to the average value of some other local property: e.g. the gradient or Laplacian might have a high average value at points in the object and a low average value at background points. To extract the object in these cases, we need only compute the gradient say at every point, obtaining a new image F' in which the object has a higher average value than the background. The

method described above can then be applied, i.e. first locally average F' to obtain a smoothed image F'' in which the object is darker than the background, and then extract the object from F'' by simple thresholding.

3.7 CONTOUR FOLLOWING.

A recurring theme in image analysis is that an image may be simplified by representing objects in the image by their outlines. The use of gradient operators has been discussed. Another technique developed for this purpose is called contour following or tracking. As the name implies, it involves tracing out the boundary between a figure and its background.

In most of the methods described thus far, the processing that was done at each point of the image did not depend on the results already obtained at other points. Thus these methods can be regarded as operating on the image 'in parallel', i.e. at all points simultaneously; and they could be implemented very efficiently on a suitable parallel computer. Contour following is a method which, in processing a point, exploits the results of previously processed points. In this inherently sequential method the processing that is performed at a point and the criteria for accepting it as a valid point can depend on information obtained from earlier processing of other points, and in particular on the nature and location of points already accepted as part of the contour. Sequential methods are so flexible and can be defined in so many different ways that a discussion of specific techniques will not be attempted as in previous sections. Rather, general classes of methods are described and suggestions of several possible variations made.

The following is a generalised raster tracking algorithm.

- 1) On the first row (or line of the raster) accept all points that meet the detection criterion. Take each such point to be the initial point of a curve C_i that is to be tracked.
- 2) On any current row other than the first row:
 - a) For each curve C_i currently being tracked, apply the appropriate tracking criterion to the points in its acceptance region; adjoin the resulting accepted point to C_i . This criterion may depend on the distance and direction of these points from the end of C_i or from some curve that extends C_i , as well as on the grey levels, contrast, etc. of these points. If no new points are accepted into C_i , the tracking of C_i has terminated. Note that a curve C_i may branch into two or more curves, in which case we must track them all; or two or more curves may merge into a single curve, in which case we need only track that one from there on.
 - b) In addition, apply the detection criterion to the points that do not belong to any acceptance region; if any points meet this criterion, take them to be initial points of new curves C_i .
- 3) When the bottom row is reached, the tracking process is complete.

Omnidirectional tracking or curve following can also be summarised in the form of a generalised algorithm.

- 1) Scan the image systematically, looking for points that meet the detection criterion. When such a point is found, it becomes the current point.

2) Examine the neighbourhood of the current point and apply an appropriate tracking criterion. As before, this criterion may depend on the grey level, contrast, etc. of the point in question, as well as its distance and direction from the current point or from some extrapolation of the curve being tracked.

a) If no points as yet unaccepted meet this criterion, tracking of the branch has terminated. In this case, we take the next point on list L (see c) as our new current point, and resume tracking. If list L is empty, go to step 1.

b) If the unaccepted points meeting the criterion all appear to lie on a single curve, accept the closest of them as belonging to the curve branch being tracked, take it as the new current point, and go to 2.

c) If these points appear to lie on more than one curve, we may conclude that the curve being tracked has branched, or has crossed another curve. In this case put all but one of the closest ones on list L for later investigation. The remaining closest point is accepted as the new current point, as in b, and go to 2.

3) When the systematic scan is finished the algorithm has terminated.

As contour following or curve tracking is intrinsically a serial operation, an error made at any step in the process makes it more likely that succeeding steps will also be in error. This should be contrasted with operations like template matching that can be performed in parallel. In their case the outcome of an operation in one part of an image does not directly affect the outcome in another

part. For this reason, the applicability of contour following seems restricted to images with low noise levels.

3.8 ENHANCEMENT.

Whenever an image is converted from one form to another, e.g. copied, scanned, transmitted or displayed, the 'quality' of the output image may be lower than that of the input. There are methods for evaluating image quality and of enhancing low quality images. Many enhancement techniques are designed to compensate for the effects of a specific (known or estimated) degradation process. This approach, generally known as image restoration, makes extensive use of filtering theory. A more elementary class of image enhancement methods is discussed here. These include methods of modifying the grey scale (e.g. by increasing contrast), deblurring, smoothing or removing noise, and correcting geometrical distortions. In all but the last of these, little or no attempt is made to estimate the actual degradation process that has operated on the image. These methods do take into account certain general properties of image degradations. For example: increasing contrast is a reasonable enhancement operation, since degradation usually attenuates the image signal: deblurring is reasonable, since degradation usually blurs and the original object or image is assumed to have been sharp: and smoothing is reasonable, since degradation usually introduces noise and the original image is assumed to have been smooth.

We may also use these enhancement techniques to make an image more acceptable and useful to its user. In general, we can use enhancement techniques to suppress selected features of an image, or to emphasise such features at the expense of other features. From

this viewpoint, enhancement can be regarded as selective emphasis and suppression of information in the image with the aim of increasing the image's usefulness. In practical situations, one must experiment extensively in order to find an effective method. In the absence of knowledge about how the given image was actually degraded, it is difficult to predict in advance how effective a particular method will be. It is often necessary to use a combination of methods, or to use 'tunable' methods whose parameters vary from place to place in the image, depending on the local context. The techniques described here provide elementary examples, and can also be used as building blocks in the design of more complex techniques.

The quality of an image depends on the purpose for which the image is intended. The image may be intended for casual human viewing, as in the case of a domestic television image, or it may be needed for precise quantitative measurement of some sort. The types and degrees of degradation that would be acceptable might be quite different in these two cases. In this section a classification of objective quality criteria is made, not criteria that involve subjective evaluation.

- 1) One class of methods for measuring image quality uses simple measures of the similarity or difference between F and G , where $G(x,y)$ is a degraded image and $F(x,y)$ is the original. A difference measure is then the mean square deviation $\iint (F-G)^2 dx dy$. Note that this does not distinguish between a few large deviations and many small ones.
- 2) Classically, image quality has often been measured in terms of the output resulting from simple input patterns such as steps and bars. One such measure is resolution, or resolving power, which describes the distinguishability in the output

of small close objects, such as sets of bars on a resolution chart. If the bar is b units wide and the spaces are equal to the bar widths, then we have $1/2b$ 'line pairs' (bar+space=pair) per unit distance. The resolution informally is the greatest number of input line pairs per unit distance such that, on the output, we can still count the bars correctly.

- 3) Another useful class of image quality measures relates to the average steepness of an edge in the output image resulting from a perfect step edge in the input. This is known as acutance.
- 4) Images are subject to many different types of noise. Some of these are independent of the image signal, but others are not. Some are uncorrelated from point to point, while others are coherent. Two types of commonly encountered noise are described below.

Channel noise: The value of this is generally independent of the strength of the image signal. In this case it can be described mathematically as $F = G + N$, where the noise N and the input image G are uncorrelated. In many cases the noise level does depend on that of the image signal. If the noise is proportional to the signal, i.e. $F = G + N \cdot G$, we then have $F = G(1 + N) = G \cdot N_1$, so that we can regard this situation as one where we have uncorrelated noise that is multiplicative rather than additive.

Quantisation noise: This is an important type of noise which occurs in digital images. It is called quantisation noise or quantisation error and it is the difference between the quantised image and the original.

3.9 TRANSFORMATIONS.

A transformation is a method for taking images from one domain to another. Transforms are of interest for two reasons they provide both a natural setting in which to investigate certain theoretical aspects of image analysis and a convenient means for implementing such operations as smoothing and template matching. The most important technique for transforming images into the spatial frequency domain is the Fourier transform. The development of the so called fast Fourier transform for digital images has increased the incentive to look closely at the utility of transform techniques in image analysis.

The Fourier transform of $F(x,y)$ can be looked upon as being merely the weighting coefficients of the expansion of F as a generalised sum of exponentials. It is important to consider in a little more detail the consequences of the definition of the Fourier transform pair, and in particular to develop the interpretation that high spatial frequencies correspond to the presence of sharp changes in the intensity of the image. In general, edges in an image introduce spatial frequencies along a line in the complex frequency plane orthogonal to the edge. The sharper the edge is, the further out one has to go from the origin along this line before the weighting coefficients become negligible. An intuitive view then is that high spatial frequencies correspond to sharp edges, low spatial frequencies correspond to the absence of edges (and therefore regions of approximately uniform grey level,) and the orientation of a spatial frequency corresponds to the orientation of an edge in the image.

The close connection between one form of template matching, as it was described earlier, is shown and the Fourier transform.

Convolution is essentially equivalent to cross-correlation and, as convolution is equivalent to multiplication of transforms, we can therefore define a template matching algorithm based on multiplying Fourier transforms and then inverting the result. Therefore the cross-correlation function of an image F with a template T is:

$$R(x,y) = F_t * T_{It}(x,y) = F^{-1}(F_t(F_x, F_y) T_{It}(F_x, F_y))$$

where $T_I(x,y)=T(-x,-y)$, T_{It} = Fourier transform of T_I ,

F_t = Fourier transform of F , F^{-1} = the inverse transform.

Although this formula involves the use of the Fourier transform three times, it is sometimes easier to do this than carry out the cross-correlation directly. The value of the cross-correlation for all translations of the template is obtained in a single operation. We could think of $R(x,y)$ as another image function whose intensity gives the degree of match of the image function with a template translated to location (x,y) . Hence, this operation is often used as a means of discovering whether a 'shape' occurs anywhere within a given image.

It was shown above that cross-correlation can be performed in the frequency domain merely by multiplying transforms. By considering the implications of transform multiplication it can be seen that there are further applications, including spatial filtering. An intuitive understanding of linear spatial filtering is probably best gained by considering some examples. A low pass filter is characterised by a transfer function $H(f_x, f_y)$ having a relatively small magnitude for frequency pairs (f_x, f_y) far from the origin of the frequency plane and relatively large magnitudes for frequencies near the origin. In other words, a low pass filter attenuates high spatial frequencies and passes low spatial frequencies. Since it has already been noted that high spatial frequencies are introduced by

the occurrence of sharp edges in the original image, we can see that when the transforms are multiplied together, low pass filtering removes sharp edges and hence produces blurred images. Low pass filtering operations, as a class, are analogous to the spatial smoothing operations discussed previously. Similarly, a high pass spatial filter is characterised by a transfer function having a relatively large magnitude for spatial frequencies far from the origin, and a relatively low magnitude for frequencies near the origin. In other words, a high pass filter attenuates low frequencies, corresponding to sharp edges, and is therefore analogous to spatial differentiation.

3.10 LINE FITTING.

Line fitting, line finding or edge detection as it is variously known is an important approach to line segmentation based on the detection of discontinuities, i.e. of places where there are more or less abrupt changes in grey level or texture, indicating the end of one region and the beginning of another. Such a discontinuity is called a line or edge.

A difficulty with edge detection as an approach to image segmentation is that the detected edges often have gaps in them at places where the transitions between regions are not abrupt enough. Moreover, edges may be detected at points that are not part of region boundaries if the given image is noisy. Thus the detected edges will not necessarily form a set of closed connected curves that surround closed connected regions. One way to overcome these problems is to use tracking techniques which follow the edges around the regions. Such techniques are designed to tolerate gaps in edges which lie on a

curve, and to reject as noise edges which do not. Another possibility is to apply some curve detection operations to the edge detector output. This rejects edge points that do not lie on curves and can also be defined to fill gaps in edges that do. A method of filling in regions that are surrounded by broken edges using a propagation process has been used. The tracking approach is usually the best, because of its great flexibility. In particular, it can be designed to take into account any information that may be available about the shapes of the regions whose boundaries are being tracked.

The global collinearity of feature points has been used as a major factor in recovering straight image edges rather than their contiguity. This is captured using a variant of the Hough transformation. A straight image edge is treated as a line in the image plane, parameterised by: α the orientation of the normal, and r , the length of the normal from the origin to the line. The equation of the line that passes through (x,y) is then

$$x\cos(\alpha)+y\sin(\alpha) = r$$

A line is thus completely specified, except for its extent, by an (r,α) pair. A point (x,y) in the image plane may be thought of as the common point of intersection of a family of lines, each member of this family having an orientation α in the range 0 to 180 degrees and a value r , determined by the equation given above. A point therefore corresponds to a family of (r,α) pairs. A set of collinear points in the two dimensional space of the image has a set of line families with a common (r,α) member. The basic principle of the technique is to accumulate evidence for such common members by mapping image points into the (r,α) space.

The image edges of interest are associated with local changes in image brightness, and thus the points which form these edges are

(x,y) locations in the image, where the gradient G of the image brightness significantly exceeds the values of G associated with locations in uniform areas of the image. It is only locations with values of G greater than a threshold that are used in the recovery of image edges. For each image location so identified, we can compute the family of (r,α) pairs it determines and begin to accumulate evidence by incrementing the appropriate counters in a two dimensional array. Each counter records the number of locations giving rise to the value of (r,α) lying within the range appropriate to that counter, a range $(dr,d\alpha)$ which is determined by the quantisation we impose on the (r,α) space. Two modifications to this technique are suggested.

- 1) Restrict the values of α to those approximately perpendicular to the direction of the gradient.
- 2) Increment the two dimensional array by the magnitude of the gradient rather than by 1.

A similar procedure has also been suggested for detecting approximate circles and approximately circular arcs of varying grey levels in an edge enhanced digital image. Three dimensions are required in this technique to accumulate evidence for centres of circles (x,y) with radius r .

3.11 OBJECT DESCRIPTION.

This section is a brief introduction to the next two sections as a number of assumptions are made in them. In image compression or enhancement, the desired output is an approximation to, or an improved version of, the input image. Another major branch of image analysis deals with image input but the desired output is a

description of the given image, or at least of parts of the image.

The description refers to specific parts, regions or objects in the image, to generate a description it is necessary to segment the image into these parts. Some segmentation operations can be applied directly to any image, while others can only be applied to an image that has been partially segmented, since they depend on the geometry of the parts that have already been extracted from the image. Probably the simplest figure extraction method of all is merely to threshold the original image. Any image point whose intensity exceeds (or fails to exceed) some threshold is declared to be part of the figure or object of interest. A generalisation of this is to partition the range of grey values into intervals and to declare all image points with intensities in a given interval to be part of the figure. A second method of feature extraction is to threshold the gradient image and thereby obtain a line like drawing. A third method is to perform a region analysis and treat each region (at least tentatively) as a distinct figure. These are only a sample of some of the segmentation techniques used to try to explain what is meant by image segmentation. The techniques are described more fully in the next two sections.

It should be emphasised that there is no single standard approach to segmentation. Many different types of image or scene parts can serve as segments on which descriptions are based, and there are many different ways in which one can attempt to extract these parts from an image. The perceptual processes involved in the segmentation of a scene by the human visual system, e.g. the Gestalt laws of organisation, are not yet well understood. For this reason, no attempt will be made to define criteria for successful segmentation. Success must be judged by the utility of the

description that is obtained using the resulting objects.

3.12 LINE DESCRIPTION.

Line description is a method of defining objects in an image by their boundaries or edges. The general approach is to fit the figure by one or more straight lines (or perhaps polynomial curves). There are two major problems; where to break the original line into segments, and how to fit a line to each segment. Most of the methods are heuristic in that they are not dignified by much supporting theory, and must be used judiciously. We begin with the second (and simpler) problem, and make the assumption that we have on hand a set of discrete points in the x,y plane to which to fit a single line.

3.12.1 Line Fitting.

Two techniques for fitting a line to a set of points are now described.

1) Minimum squared error line fitting.

Given a set of points (x_i, y_i) , $i=1, \dots, n$ in the plane, we seek two numbers C_0 and C_1 such that the error function $\sum_i ((C_0 + C_1 x_i) - y_i)^2$ is minimised. In other words, we want to find the straight line such that the sum of squares of the vertical distances from each point to the line is a minimum. This problem and indeed a generalisation of this problem can be elegantly solved by the use of the pseudo-inverse of a matrix.

Suppose we have a matrix A and A' , i.e. the pseudoinverse of A , $= A^{-1}$, i.e. the inverse of A . On the assumption that the columns of A are linearly independent, we know that $\|Au - b\|^2$ has a minimum value if we take $u =$

A'b. Suppose we take our set of points (x_i, y_i) and write the following matrix equation, of the form $Au = b$, i.e.

$$\begin{array}{cc} \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 & x_n \end{bmatrix} & \begin{bmatrix} C_0 \\ C_1 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \cdot \\ \cdot \\ y_n \end{bmatrix} \\ A & u \quad b \end{array}$$

If we take C_0, C_1 to be $A'b$, then we are assured that $\|Au - b\|^2$ is minimised. Also $\|Au - b\|^2 = \sum_i^N ((C_0 + C_1 x_i) - y_i)^2$ which is precisely the minimum squared error criterion for the coefficients C_0 and C_1 of the best line. Hence the minimum squared error solution is obtained by multiplying the vector b of the y values by the pseudoinverse of A , the matrix of x values. $(C_0, C_1) = A'b$.

2) Eigenvector Line Fitting.

Another method for fitting a straight line to a set of points can be derived if the definition of 'best' fit used in the minimum squared error approach is slightly altered, i.e. by defining the line to be the best fit to a set of points if it minimises the sum of squares of the perpendicular distances from the points to the line. For reasons that will become apparent, this line is called the best eigenvector fit. To illustrate the distinction (see figure 6), the minimum squared error method would select the line minimising the sum of squared lengths of the vertical solid lines, whereas the method discussed in this section would minimise the sum of squared lengths of the

dotted lines. Thus the eigenvector fit, unlike the minimum squared error fit, is not a function of the choice of axes.

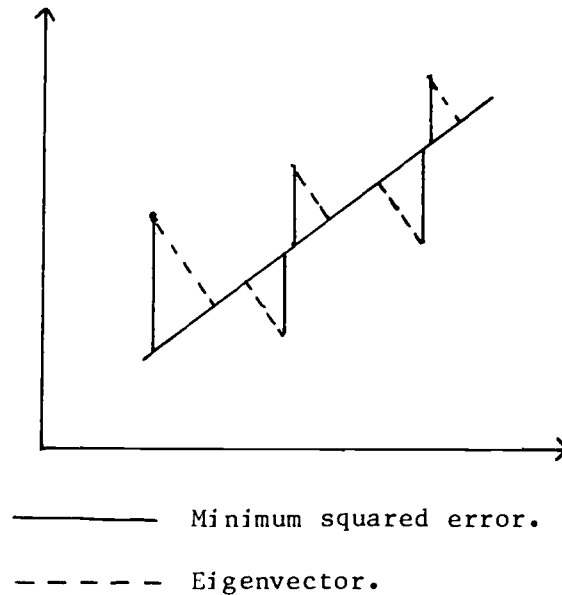


Figure 6. Two Best Fit Criteria.

We assume that we have a set of n points (x_i, y_i) , $i=1, \dots, n$ to which we want to fit a straight line. We denote the i th point (x_i, y_i) as the vector V_i , and let d_i be the perpendicular distance from V_i to the line. Our task is to find the line minimising

$$D^{**2} = \sum_i (d_i^{**2}).$$

The derivation of the best line is considerably simplified if we first establish the fact that any line minimising D^{**2} must pass through the mean (or centre of gravity) of the points (x_i, y_i) . Suppose we have found the line minimising D^{**2} . We set up a new coordinate system, the U-W system, such that the W axis is parallel to the best line. Let (u_i, w_i) be the U-W coordinates of (x_i, y_i) . In the U-W system the equation of the best line is $u_i = u_0$, for some u_0 , and the squared distance from the i th point to the best line is

$(u_i - u_0)^2$. D^2 is minimised if u_0 is the U coordinate of the mean of the n points. The best line therefore passes through the mean and, since the mean does not depend on the coordinate system, our initial assertion is proved.

In view of the preceding argument we may as well assume that the set of n given points has zero mean. Our problem then becomes: given a set of n points with zero mean, find the best line passing through the origin such that D^2 is minimised. Suppose that we characterise a line through the origin by its unit normal vector N . Then, explicitly denoting the dependence on N , we have:

$$d_i^2(N) = (N \cdot V_i)^2 = (N^t V_i)^2$$

and

$$\begin{aligned} D^2(N) &= \sum_i^n (d_i^2(N)) \\ &= \sum_i^n ((N^t V_i)^2) \\ &= \sum_i^n ((N^t V_i)(V_i^t N)) \\ &= N^t \sum_i^n ((V_i V_i^t) N) \end{aligned}$$

The symmetric matrix $S = \sum_i^n (V_i V_i^t)$ is the scatter matrix of the n given points. The best line is therefore characterised by the unit normal vector N such that it minimises $D^2(N) = N^t S N$.

This quadratic form is minimised, subject to $N^t N = 1$, by taking N to be the eigenvector of S associated with the smallest eigenvalue. Since distinct eigenvectors of symmetric matrices are orthogonal, the best fitting line is in the direction of the principal eigenvector of S , that is the eigenvector associated with the largest eigenvalue. The prescription then for the best fitting line is:

- a) Standardise the points by subtracting the mean of the set from each point.
- b) Find the principal eigenvector of the scatter matrix of the set of standardised points.
- c) The best fitting line is the unique line through the mean of the set of points which is also parallel to this eigenvector.

3.12.2 Line Segmentation.

In some situations the problem of partitioning the figure into subsets can be thought of as a segmentation problem. There might, for example, be reason to expect the figure points to lie on some (possibly complex) curve, rather than on random line segments. In these situations it is natural to look for techniques which partition the figure points, or the curve, into a number of consecutive line segments such that each segment can be reasonably approximated by a straight line. The following discusses two methods for doing this.

1) Iterative end point fits.

The method of iterative end point fit in its simplest form proceeds as follows. Given our usual set of n points, we fit an initial line, AB say, by merely connecting the end points of the set. The distances from each point to this line are computed and, if all the distances are less than some preset threshold, the process is finished. If not, we find the point furthest from AB , C say, and break the line into two new lines AC and CB . This process is repeated separately on the two new lines possibly with different thresholds. The process is illustrated in figure 7 below.

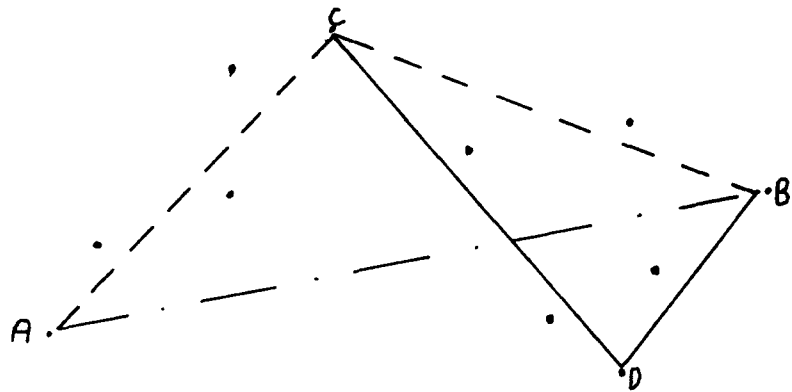


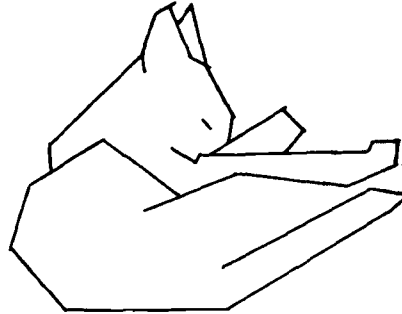
Figure 7. Iterative end point fit.

The method has two drawbacks, both of which can be traced to the fact that it can be strongly influenced by single points. The first, and minor drawback, is that a segment finally selected may not be a particularly good fit to the points in its immediate vicinity. This problem can be alleviated by a post fit process that uses a more sophisticated algorithm to adjust the position of the line segments. The iterative algorithm can be used to determine an approximate fit and approximate positions of the break points these approximations can then be improved during a second pass. The more serious drawback of the iterative process is that a single 'wild' point can drastically change the final result. While this problem can also be alleviated, by a preliminary smoothing process, the same basic problem is encountered by many other image analysis algorithms. Simplicity is often obtained at the price of basing decisions on purely local image information. The applicability of simple algorithms, therefore, is

generally restricted to data that is initially largely noise free.

2) Points of maximum curvature.

Consider here a different kind of segmentation problem. Given a smooth contour (which might be obtained for example from a contour follower), what is a reasonable way to segment the curve into a series of straight lines? One suggestion based on psychological experiments is to break the curve at points of high curvature and connect the breakpoints by straight lines. The validity of this approach is attested to by a famous example first suggested by Attneave (ATT54), shown in figure 8 which is constructed according to this rule. Few people have difficulty in recognising it as a sleeping cat. Mathematically this approach is related to the so called intrinsic equation of a curve, defined to be its curvature as a function of its length. Hence, the method under discussion is essentially equivalent to representing the intrinsic equation by its end points.



From Attneave 1954.

Figure 8. Points of maximum curvature.

3.12.3 Chain encoding.

A very convenient method for representing an arbitrary curve is known as chain encoding. For purposes of simplicity, assume that initially the curve is represented as a black-white image on the unquantised plane and that it is required to represent it digitally in some manner. Rather than digitise the entire image in the conventional manner, it is possible to place a mesh over the analogue image and identify those points where the curve crosses some line of the mesh. The vertices of the mesh nearest each intersection are taken to represent the curve. We then encode the vertices in an octal sequence by giving the direction from one vertex to the next in accordance with the code shown below. The code specifies angle (as opposed to change of angle) as a function of line length, it being understood that diagonal lines are longer than vertical and horizontal ones by a factor of $\sqrt{2}$. Chain encoding is particularly useful for comparing the shape of two curves. This is done by calculating

the chain cross-correlation.

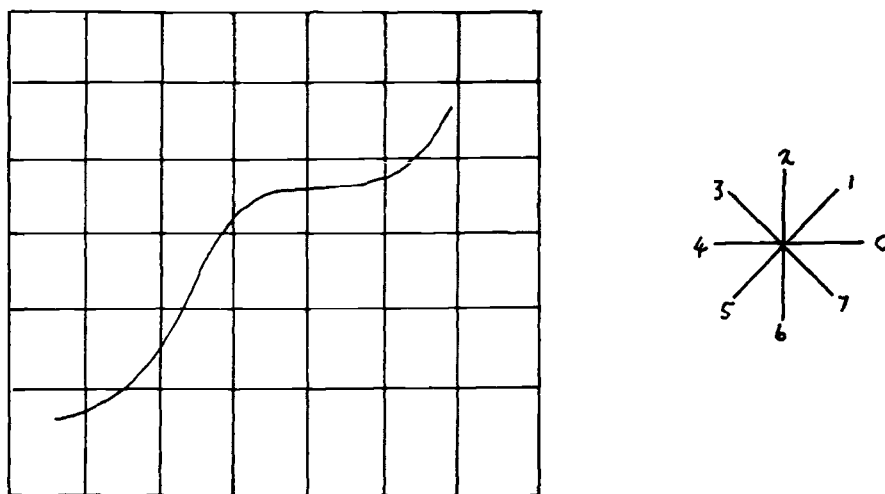


Figure 9. Chain Encoding.

3.13 SHAPE DESCRIPTION.

Turning now to the problem of describing the shape of a figure. Assuming the figure has been extracted from the background, as with line description, the problem of characterising subsets of the plane is dealt with. Before launching into a discussion of specific methods, let us digress briefly and consider a little more fully the notion of a description. A mathematically complete description would specify for every point in the image whether or not that point was a member of the subset. Completely specifying the points in the subset does no credit to our intuitive notion that a description of a complex thing should be simpler in some sense than the thing being described. An alternative method of describing a figure depends on recognising it as a member of a previously described class. If the figures of interest are in fact simple, easily recognisable classes, then this approach can be employed. Our primary interest is in

methods that lead to simpler characterisations of figures than enumeration of points, but do not presuppose the recognition of a figure as a member of some class.

Even after excluding very explicit descriptions of the type referred to above, wide latitude is available in our choice of characterisation. Broadly speaking, our choice depends upon how 'informative' the description is required to be. While very informative descriptions have the advantage of precision, they also have several significant disadvantages. Perhaps the most important is based on the fact that it is often convenient to think of certain families of figures as being indistinguishable from the figure itself. If we restrict ourselves to very informative descriptions, the descriptions of a given figure and one of its translations would be different. Other often used equivalence classes of a given figure are the sets of its rotations and dilations. In general, we speak of descriptions that are invariant under certain classes of transformation. The challenge is to find descriptions that are invariant to transformations leaving figures unaltered in 'unimportant' ways, and yet sensitive to transformations that change figures in 'important' ways.

1) Topological properties.

A fundamental way of characterising a subset of the image is to specify some of the topological properties of the set. A topological property is a property that is invariant to so called rubber sheet distortions. Put graphically, if the image were represented by a rubber sheet, then a topological property of a subset of the sheet would not be changed by any stretching deformation of the sheet. (A topological property would in general be changed by ripping the sheet or fastening part of the

sheet to itself.) A rubber sheet distortion of the image is called a topological mapping or homeomorphism of the image onto itself. Notice that topological properties of sets cannot involve any notion of distance, since distances are distorted by topological mappings. Similarly they cannot involve any properties that ultimately depend upon the notion of distance, such as the area of a set, parallelism between two curves, perpendicularity of two lines and so on. Obviously topological descriptions of sets will be very general indeed and thus will not be very informative in the sense discussed above.

One commonly used topological description of a set is the number of connected components. A connected component of a set is a subset of maximum size such that any two of its points can be joined by a connected curve lying entirely within the subset. A second topological property of interest is the number of holes in the figure. Formally the number of holes in a figure is one less than the number of connected components in the complement of the figure. Topological descriptions of figures find application in image analysis as a preliminary sorting parameter, as a check on the accuracy of other descriptions, and as an adjunct to other descriptions. In general a problem in which topological descriptions alone suffice is unusual.

2) Metric properties.

Conceptually metrics are generalisations of Euclidian distance, so a metric property will in general be changed if the image plane undergoes any distortion. This being the case, we might anticipate that metric properties will often lead to more informative figure descriptions than the topological properties discussed earlier.

A metric D is a real valued function of two variables in the image such that for all points x, y and z in the image the following properties are satisfied:

- a) $D(x, y) \geq 0$ and $D(x, y) = 0$ if and only if $x = y$ (positivity)
- b) $D(x, y) = D(y, x)$ (symmetry)
- c) $D(x, y) + D(y, z) \geq D(x, z)$ (triangle inequality)

The most common metric is Euclidean distance. Given $X = (x_1, x_2)$ and $Y = (y_1, y_2)$

$$D(X, Y) = \text{SQRT}((x_1 - y_1)^2 + (x_2 - y_2)^2).$$

Also useful in image analysis are:

$$\text{Absolute value metric } D_a(X, Y) = |x_1 - y_1| + |x_2 - y_2|.$$

$$\text{Maximum value metric } D_m(X, Y) = \text{MAX}(|x_1 - y_1|, |x_2 - y_2|).$$

Care should be taken in choosing metrics as not all appealing functions are metrics. A list of commonly used metrics are given below.

- a) Area A .
- b) Length of perimeter P .
- c) Thinness ratio $T = 4\pi(A/P^2)$.
- d) Aspect ratio $AS = L/W$. L = length, W = width.

3) Descriptions based on irregularities.

Continuing with metric properties, there are two descriptive methods designed to make explicit the significant 'irregularities' of figures. The first exploits deviations from convexity, and the second capitalises on local extrema of the perimeter. Convex hull and convex deficiency is what the first technique is called. A convex set is a set containing every line segment that connects two of its points. The convex hull H of a set S is the smallest convex set containing S . If S is convex to begin with then of course $H = S$. If S happens to have only one

connected component, then H can be visualised as the set enclosed by a rubber band stretched around the perimeter of S . The set difference $H-S$ is called the convex deficiency D of the set H .

Local extrema of a figure boundary can also be used to characterise a figure. We can describe the boundary of a figure by the points at which it achieves a local extremum in either the X or Y directions. As it happens these points are often near the points of high curvature of the boundary although this is not a universal situation. If figure description by extreme points is to be feasible, it is clearly necessary to smooth out minor fluctuations in the figure boundary. Also, in general, an extrema description is sensitive to the orientation of the figure with respect to the coordinate axes.

4) Skeletons of a figure.

A most intriguing way of characterising a figure is provided by the medial axis or prairie fire transformation. Its purpose is to extract from the original figure a stick-figure-like representation aptly called a skeleton. Moreover the transformation also extracts supplementary information, that together with the skeleton, permits the original figure to be reconstructed.

There are many equivalent definitions of the skeleton of a figure. Perhaps the most intuitive is the following. Imagine that the interior of the figure is composed of dry grass and that the exterior, or background, of the figure is composed of unburnable wet grass. Suppose a fire is lit simultaneously at all points along the figure boundary. The fire will propagate at uniform speed to the middle of the figure. At some points,

however, the advancing line of fire from one region of the boundary will intersect the fire front from some other regions and the two fronts will extinguish each other. These points are called quench points of the fire. The set of quench points defines the skeleton of the figure. Since the fire propagates at uniform speed, a quench point must be equidistant from at least two boundary points.

5) Analytical descriptions of shape.

There are several ways in which an arbitrary figure can be represented, in principle at least, exactly. The most straightforward such representation merely specifies for each point in the image whether that point is in the figure. This representation is called the characteristic function of the figure; the characteristic function is zero for points outside the figure and one for points inside the figure. A second exact representation of a figure can be based on the intrinsic equation that specifies the boundary curvature as a function of the arc length measured from some arbitrary boundary point. These complete figure descriptions run counter to our notion that a description should be simpler than the object being described. However, various mathematical methods can be used to approximate these complete descriptions and thereby obtain simpler ones. The standard approach is to form a series expansion of an exact representation and then only use the first few terms of the series.

6) Integral geometric descriptions.

Integral geometry is an established mathematical discipline concerned with calculating the probabilities of various kinds of random geometrical events. Its application to

the problem of describing figures has the following general flavour. Suppose we are given a single component, simply connected figure. Imagine an experiment in which random lines are tossed at the retina on which the figure is displayed. For the sake of simplicity, suppose that we are interested in two different aspects of the experiment; first, in the proportion of random lines that intersect the figure, and second, the average length of the chord cut off by the figure on those lines that do intersect. These properties and many others like them, are strongly dependent on the given figure and hence may be used for descriptive purposes.

A few results are given here.

- 1) The probability that a random line intersects the figure is H/R , where H is the length of the perimeter of the convex hull of the boundary B and R is the length of the perimeter of the retina.
- 2) The expected number of times that a single line intersects B is $(2B)/R$ where B is the length of the boundary.
- 3) The expected length of the intersection of a random line with an arbitrary figure is proportional to the area of the figure.

4. BLIPS SYSTEM DESCRIPTION.

4.1 Introduction.

4.2 System Objectives.

4.3 System Structure.

4.4 Worked Examples.

4.5 Evaluation of Techniques.

4.1 INTRODUCTION.

This chapter describes a package for use in the design of image analysis systems; it has been implemented and developed by the author. The principal aims and objectives of the package are dealt with in section 4.2, a description of the structure of the package is given in section 4.3, a number of examples of the effect of different image analysis techniques are given in section 4.4 and finally a summary of the effectiveness of these different techniques on different types of data is given in section 4.5.

Some of the capabilities of this package are not easy to distinguish from the capabilities of the computer system on which it is implemented. This is because the computer requirements of the package and of image analysis in general are fairly specific in certain areas, e.g. graphical output, on-line interaction, the use of large data areas. We can see then that some of the package's

facilities are only possible to implement as a result of the facilities available on the computer system used, in this case the Edinburgh Multi-Access System, EMAS (WHI73). However, a description of the BLIPS (Bill Laing Image Processing System) package as if it itself had all the attributes which might normally be considered system attributes is given. This seems reasonable as it would not be possible or useful to implement the package on a system where these facilities could not be exploited.

The description of BLIPS given in this chapter is at a high level, in the sense that it is about the design objectives, the general structure and the capabilities of the system. It is not detailed enough to allow someone to use the package or modify it by including more algorithms. The intention here has been to give a flavour of the philosophy behind its design and implementation. A full user guide to the package, which gives a more detailed specification, is given in Appendix 1.

4.2 SYSTEM OBJECTIVES.

The BLIPS package has been designed to work primarily in an interactive environment although it can be used in batch mode. The principal aim of the package is to allow an image analysis system designer to experiment with and evaluate the effects of different techniques on his data. This is an important requirement because the design of image analysis systems has no theoretical basis. There are no reference works to which the designer can turn to help him. In fact, it appears that different methods and techniques are necessary for different types of data, different 'qualities' of data, and even different applications using the same data. The major conclusion that

can be drawn from this is that currently the development of image analysis techniques is influenced primarily by the data rather than theoretical principles. As a result, it is important for the designer to be able to familiarise himself with his data: the BLIPS package helps him to do this.

Control of the package is through a simple command language consisting of verbs, data and delimiters. The addition of new algorithms and commands to the package is straightforward. It is therefore easy to develop the package in the rapidly changing environment of image analysis. Output from the package in textual form can either be directed to an interactive terminal or to a print file. Graphical output can either be generated in a form suitable for a raster plotter, i.e. a matrix plotter, or a conventional graph plotter. A feature of image analysis is the requirement to handle large arrays of data often in a random order. This package is as a result most suitably implemented on a computer with a large address space or virtual memory support, so that the quantity and accuracy of the data is not unnecessarily restricted. The user can therefore apply many algorithms to his data before outputting it to backing store or in a graphical format to a plotter.

4.2.1 Specific Objectives of the Package.

Below, a list of the specific objectives of the BLIPS package is given. These are general design aims and in fact are mainly a statement of the philosophy behind the design of the package.

- 1) The package is problem-orientated.
- 2) It is an attempt to provide a 'bag of tools' to deal with a 'bag of problems'.

- 3) It is not a specific image analysis solution but an attempt to allow the designer of such a system to interactively choose the techniques which suit his data best.
- 4) The key to successful image analysis does not lie with one particular technique but with the application of the 'correct' technique to specific data.
- 5) It allows feedback between the designer and his data.
- 6) It puts the 'human' in charge of the design and evaluation process.
- 7) It makes available known procedures to the image analysis system designer.

4.2.2 General Capabilities of the Package.

Below, a list of the general capabilities of the BLIPS package is given. These are in fact capabilities that any package should have which is interactive by design and which allows the user to apply different algorithms to arbitrary data.

- 1) The ability to communicate with and control the package through simple procedures. This is necessary to provide an ergonomic interface to the user.
- 2) A quick response to user commands or requests especially for simple procedures. This is absolutely essential if the user is to 'interact' with the package.
- 3) An ability to go backwards or forwards to any option available in the package. This is again an essential requirement for any package which purports to be interactive.
- 4) An ability to generate and modify on-line algorithms and

programs. This is necessary to allow the rapid development and refinement of algorithms.

5) A straightforward method for selecting, labelling, merging and splitting of data sets. This allows the user to manipulate his data and also to leave a partially modified dataset for further processing at a later date.

6) An ability to access large amounts of data and large programs without difficult memory management problems.

The BLIPS package was designed to have these capabilities. Its utility as a method for designing an image analysis system is discussed later in this thesis. However, I would like to stress now that having these capabilities has made it easy to use and develop.

4.3 SYSTEM STRUCTURE.

The BLIPS (Bill Laing Image Processing System) package is mostly written in the high level language IMP (STE75) which was developed at the University of Edinburgh for systems programming and general purpose scientific programming. The package can and does make calls on external procedures written in other high level languages, e.g. FORTRAN and ALGOL60. As an example, the fast Fourier transform algorithm currently used by the package is the one written by the Numerical Algorithms Group, which is in use in many universities and scientific establishments. In fact all of the other algorithms have been written by the author but subsequent developments would certainly include adding algorithms acquired from other sources. As the package is primarily designed to be interactive, although it can be run in a batch mode the description of its use in the remainder of this chapter will be of its interactive use.

The data, i.e. images, for input to the package are kept in partitioned data files with each image in a separate partition. Images produced or 'saved' by the package are generated in the correct format to be included in a partitioned file for subsequent input to the package. The output from the package is directed to the interactive terminal, although graphical output can be sent to a graphics terminal, a matrix plotter, a graph plotter or a line printer.

Commands to the BLIPS package have three components; a verb, an optional number of parameters and a delimiter. The verb is supplied first, optionally followed by the parameter(s) in brackets and terminated by a newline character or a semicolon.

e.g. SHADE(0,100)

GET(PIC1) ; RANGE ; PRINT(TEMP)

The semicolon simply allows more than one command to be typed on each line. Input images to the package must have the format NUMBER OF LINES, LINE LENGTH, followed by the image data supplied line by line starting at the top left in the image. Images are 'named' objects in the partitioned file,

e.g. partitioned file PICTURES can contain the images named

THERM1, THERM2, CIRCLES, etc.

Conceptually the images are 'got' from the partitioned file into what the package regards as the current image which is available for manipulation. For example, the image may be transformed into the gradient image by applying a gradient operator to it. Any other operator can then be applied to the gradient image and so on.

The parameter part of the command need not be supplied, or may comprise integers, strings, or a combination of the two. The defaulting of parameters to standard values and the optional omission

of parameters is currently not available. Each algorithm or procedure is written as a separate program which is then linked with the main control routine when the package is run. This has the advantage of allowing the user to change a particular algorithm without having to recompile or of having complete knowledge of the whole package. The main control routine handles the input and output of images, the interaction with the user, the validation of parameters and the calling of the algorithms and procedures. The controlling routine checks the type of the parameters input by the user before calling the appropriate routine which is then left to check the range and validity of the parameters. It is very straightforward to add a new algorithm to the package. First the user writes his algorithm in the programming language of his choice, conforming to the restrictions of the cross-calling mechanism on the computer system. In the current implementation of the package, the programming languages used are ALGOL60, FORTRAN and IMP. The user then adds to the controlling routine the name of the new algorithm, a specification of its parameters, whether it requires a current image to be available or not, and a call of the new routine. All of this is explained more fully in Appendix 1.

The most important concept for the user to understand is that the package provides him with a 'retina' or current image, into which images from backing store can be written, to which transformations can be applied, from which measurements can be taken, from which images can be transferred to backing store and which can be printed or displayed graphically. As far as the user is concerned this appears as an integer array except in the case of an image after a Fourier transform has been applied. The current image in this case is stored in two arrays, one representing the real part and the other

the imaginary part of the transform. To apply subsequent operations to this image an inverse transform must first be applied unless the user wishes to manipulate the real and imaginary arrays or to display the transformed image. In these cases special routines must be used. One is currently available which displays the magnitude of the real and imaginary parts of the Fourier transform.

The currently available commands to the package can be grouped into three types.

The first type is managerial or input-output commands,

e.g. GET(IMAGE), PRINT(FILE), SIZE, RANGE, etc.

The second type is display or graphics commands,

e.g. SHADE(0,50), SHADEFourier, CONTOUR(0,10,100), etc.

The third type is the computational or manipulative algorithms,

e.g. SCALE(50,100), FOURIER, THRESHOLD(100,150), etc.

A full list of the commands currently available and their parameter specifications is given in appendix 1.

Although the user of the image analysis system is primarily interested in the computational algorithms for image processing, feature selection and pattern analysis that are available on the system, the system architecture is what ultimately determines the nature and extent of its interactive capability. This is principally what has so far been described in this chapter.

4.4 WORKED EXAMPLES.

This section examines the effect of different image analysis techniques on some artificial data or phantoms. The reason for examining the effect of different algorithms on phantom data is that it is often easier to see and to explain the resulting effects and in

fact it may also be easier to quantify the effects. Two sets of data are presented here, i.e. an image containing two disks of different intensities and the same image with noise added. In all ten different algorithms are applied to each set of data giving twenty different results in all. These results can be seen in figures 10 to 29 inclusive, at the end of this chapter. As can be seen from the number of results mentioned above it is impossible to give an exhaustive representation of even a few of the different phantom images which have been used in the course of this research. For example ten different images plus the same images with noise added gives twenty different images and applying only ten algorithms would give two hundred different images. Consequently only one phantom image with and without noise added is presented here.

The type of noise which has been added to the image in the phantom set is what is commonly known as salt and pepper noise. This type of noise is randomly spread throughout the image and affects the image points by a random amount. More generally we can apply the term salt and pepper noise to any situation in which scattered points of an image are markedly darker or lighter than their immediate surroundings. Smoothing or averaging is often used to combat this type of noise.

In figures 10 to 29 presented at the end of this chapter the phantom image with and without noise added is shown and the effects of the different techniques applied to the images are shown. Below I give an example of using the BLIPS system. It has been used in this instance to produce these results by applying the following techniques, thresholding, simple differentiation using the square root, simple differentiation using moduli, a Laplacian, a smoothing differentiation operator, a circle finder and a contour follower.

A worked example of the using the BLIPS package. The text on the right of the page is a commentary on the dialogue with the package reproduced on the left.

BLIPS(PHANTOMS)

Call the package with the partitioned data file containing the phantoms as a parameter.

GET(CIRC2)

Get the phantom image CIRC2 which is contained in the partitioned file PHANTOMS into the current image.

RANGE

Find the range of values in the image CIRC2.

MAX 221 at line 15 col 68

The package outputs the maximum and minimum values and their positions.

MIN 0 at line 1 col 1

SHADE(0,256)

Output a shaded version of the image to the matrix plotter. The first parameter specifies the value of white and the second the value of black. See figure 10.

THRESHOLD(0,50)

Map the values in the range 0 to 50 into the value 1 and the values outside this range into the value 0.

SHADE(0,1)

Output the result. See figure 11.

GET(CIRC2)	Get the phantom CIRC2 into the current image again as the thresholding will have changed the current image.
THRESHOLD(100,256)	Map the image values in the range 100 to 256 into 1 and the others into 0.
SHADE(0,1)	Output the result see figure 12.
GET(CIRC2)	Get the phantom again.
ROBERTS	Apply the Roberts cross operator to the current image.
RANGE	Find the range of the resulting values.
MAX 313 at line 14 col 68	The package outputs the range.
MIN 0 at line 1 col 1	
SHADE(0,400)	Shade the resulting image with 0 as white and 400 as black. The result is shown in figure 13.
GET(CIRC2)	Get the phantom again.
ROBERTM	Apply the more efficient Roberts cross operator.
RANGE	Find its maximum and minimum values.
MAX 442 at line 14 col 68	The package outputs the range.
MIN 0 at line 1 col 1	
SHADE(0,400)	Shade the result see figure 14.
GET(CIRC2)	Get the phantom again.

SOBEL	Apply the SOBEL operator.
RANGE	Find its maximum and minimum.
MAX 988 at line 14 col 66	The package outputs the range.
MIN 0 at line 1 col 1	
SHADE(0,1000)	See figure 15.
GET(CIRC2)	Get the phantom again.
LAPLACE	Apply the Laplacian operator.
RANGE	Find the resulting range.
MAX 584 at line 52 col 61	The package outputs the range.
MIN 0 at line 1 col 1	
SHADE(0,1000)	See figure 16.
GET(CIRC2)	Get the phantom again.
CONTOUR(0,20,240)	Display on the graph plotter the contours starting at level 0 and increasing in increments of 20 to level 240. See figure 17.
CIRCLES(20,10,45)	As contour following does not change the current image it can be used immediately. This operator tries to find circles of radius 20 using a threshold of 10 and searching within an angle of 45 degrees.
RANGE	Display the range found.
MAX 3 at line 36 col 69	The package outputs the range.
MIN 0 at line 1 col 1	
SHADE(0,20)	See figure 18.
GET(CIRC2)	Get the phantom again.

CIRCLES(30,5,45)	As above but using different parameters.
RANGE	Find the range of values.
MAX 4 at line 36 col 69	The package outputs the range.
MIN 0 at line 1 col 1	
SHADE(0,20)	See figure 19.

The same sequence was repeated for the same phantom with salt and pepper noise added to 10 percent of the image points.

GET(CIRC2N)	Get the phantom image CIRC2N which is contained in the partitioned file PHANTOMS into the current image.
RANGE	
MAX 255 at line 25 col 99	
MIN -113 at line 39 col 15	
SHADE(0,256)	See figure 20.
THRESHOLD(0,50)	
SHADE(0,1)	See figure 21.
GET(CIRC2N)	Get the phantom again.
THRESHOLD(100,256)	
SHADE(0,1)	See figure 22.
GET(CIRC2N)	Get the phantom again.
ROBERTS	
RANGE	
MAX 356 at line 24 col 99	
MIN 0 at line 1 col 1	
SHADE(0,400)	See figure 23.

GET(CIRC2N)	Get the phantom again.
ROBERTM	
RANGE	
MAX 503 at line 24 col 99	
MIN 0 at line 1 col 1	
SHADE(0,400)	See figure 24.
GET(CIRC2N)	Get the phantom again.
SOBEL	
RANGE	
MAX 998 at line 22 col 87	
MIN 0 at line 1 col 1	
SHADE(0,1000)	See figure 25.
GET(CIRC2N)	Get the phantom again.
LAPLACE	
RANGE	
MAX 1020 at line 38 col 14	
MIN 0 at line 1 col 1	
SHADE(0,1000)	See figure 26.
GET(CIRC2N)	Get the phantom again.
CONTOUR(0,20,240)	See figure 27.
CIRCLES(20,10,45)	
RANGE	
MAX 4 at line 23 col 55	
MIN 0 at line 1 col 1	
SHADE(0,20)	See figure 28.
GET(CIRC2N)	Get the phantom again.
CIRCLES(30,5,45)	
RANGE	
MAX 3 at line 14 col 55	

MIN 0 at line 1 col 1

SHADE(0,20)

See figure 29.

The images produced by these steps are shown in figures 10 to 29 reproduced at the end of the chapter. The effectiveness of the different techniques is discussed in the next section.

4.5 EVALUATION OF TECHNIQUES.

The evaluation of these techniques is very subjective and the decision about whether one technique is better than another can in fact only be based on the utility of the result of the technique for subsequent processing. Nevertheless it is possible to make a few general statements about each of the techniques and the effect of noise on the techniques.

Dealing now with each technique in turn an assessment of each technique referring to the figures reproduced at the end of the chapter is given.

1) The Original Image. Figures 10,20.

The image consists of two disks of different intensities on a white background. One of the disks has a radius of 30 pixels and the other has a radius of 20 pixels. The noise which has been added to the image effects 10 percent of the image points and modifies each of these points by adding or subtracting a random intensity.

2) Thresholding. Figures 11,12,21,22.

The thresholding technique used maps the points in the image which are in a specified range into 1 and all other points into 0. Two examples of thresholding are shown one

which maps all points below a value into 1 and the other which maps all points above a value into 1. This technique is very successful on the image which has no noise added. It successfully picks out the background in one case and the figure in the other. Clearly when noise is added to the image distinguishing the figure from the background is not so successful.

3) Roberts. Figures 13,23.

This technique is a simple differentiation algorithm often called the Roberts cross operator after its first published use in image analysis (ROB65). It works simply by finding the difference between two adjacent image points, squaring the result, adding this to the square of the difference between two other adjacent image points in an orthogonal direction and taking the square root of the result. It is very effective at distinguishing the edge of the disks especially in the image with no noise added. It can be seen that the edge between the two disks is not distinguished quite so clearly as the edge between the disks and the background. In the image with noise added the edges are not distinguished nearly so clearly. Obviously some smoothing would need to be carried out on the image or a very intelligent curve follower, not distracted by spurious points, would need to be used to pick out the edges.

4) Robertm. Figures 14,24.

This technique is a more efficient implementation of the Roberts cross operator. The sum of the moduli of the differences are used rather than the squares and square

root. In fact qualitatively it gives a very similar result to the Roberts operator.

5) Sobel. Figures 15,25.

This technique is an example of a smoothing and differencing operator. When applied it smooths the effects of noise during the differencing. From the two figures reproduced it can be seen that it clearly picks out the edges, but in particular in the noisy image it tends to reduce the effect of the noise and only enhance the true edges.

6) Laplacian. Figures 16,26.

This technique is another edge enhancing technique. From the images reproduced it can be seen that it is moderately successful when no noise is present but in the presence of noise the edges are weakened and are in fact not as strong as some of the edges caused by noise points.

7) Circle Finding. Figures 18,19,28,29.

This technique attempts to identify the centres of circles of a fixed radius in an image by accumulating evidence about possible circles which would pass through the edge points of the circles. Two examples are given one searching for a circle of radius 30 and the other for a circle of radius 20. Leaving the effect of noise on the technique for the moment, it can be seen that the technique is not very succesful in finding the circle centres although the general location of the most likely point is correct. Clearly in this example the overlapping of the disks severely effects the utility of the technique. When noise is added to the image the technique

becomes completely worthless.

8) Contour Following. Figures 17,27.

This technique outlines the points in the image greater than a specified intensity. The two figures in fact show many contours drawn on the same image points. Clearly when no noise is added the contour follower successfully outlines the disks in the image. When noise is present it does moderately well in distinguishing the disks from the background but not the overlap between the disks.

To summarise the effectiveness of these different techniques on one particular set of data is not easy. In fact the principle conclusion that I have come to about different image analysis techniques is that they are most strongly effected by the different data on which they operate. It is very difficult and very dangerous to generalise about techniques and it is also difficult to predict how techniques will work on different data. In fact empirical evaluations and ad hoc techniques appear to be very successful. The most important point then is to try to understand the underlying reason why the different techniques work. In this this chapter I have only really covered some differentiation and edge enhancement techniques but from this it can be seen that some techniques are more successful than others especially in the presence of noise. Apparently the most succesful technique with this phantom image data was the SOBEL or smoothing and differencing operator as it was most successul in detecting the disk edges even in the presence of noise.

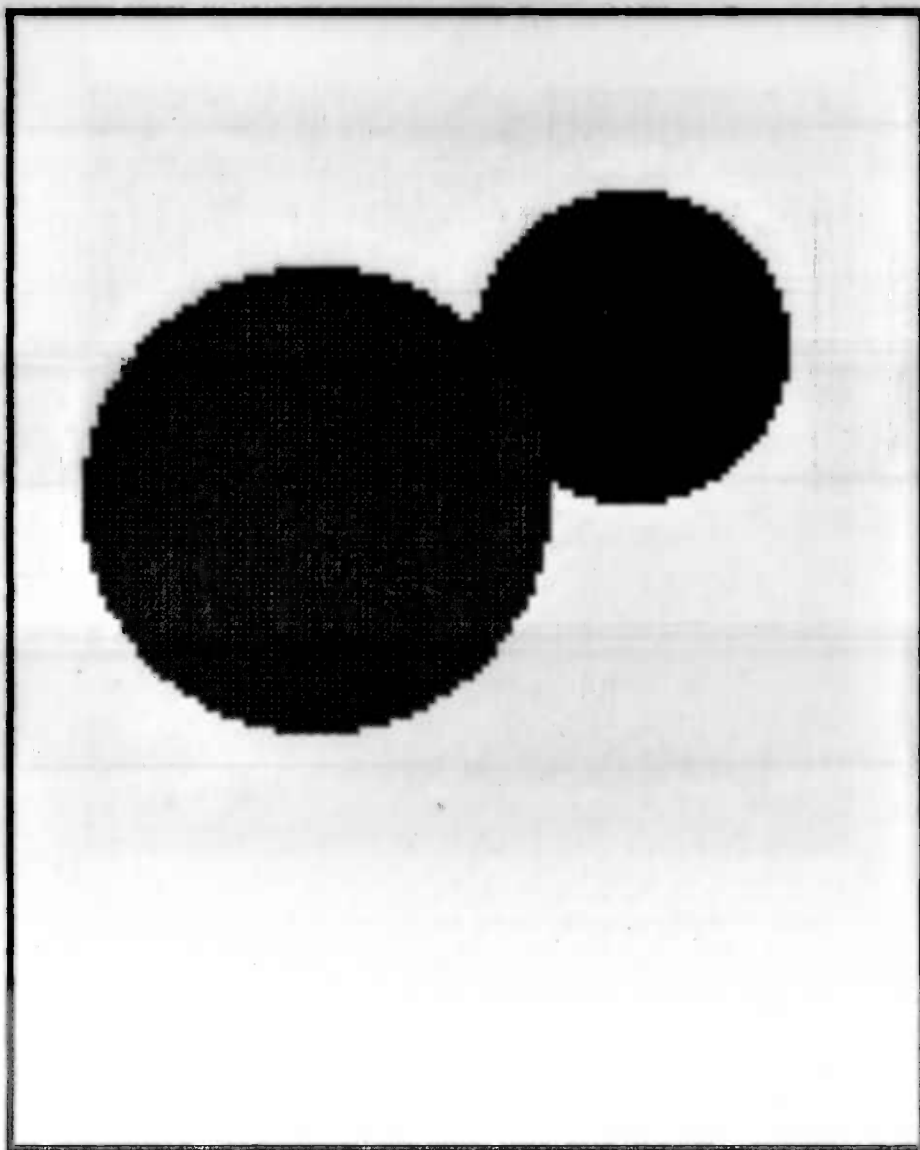


Figure 10. CIRC2.

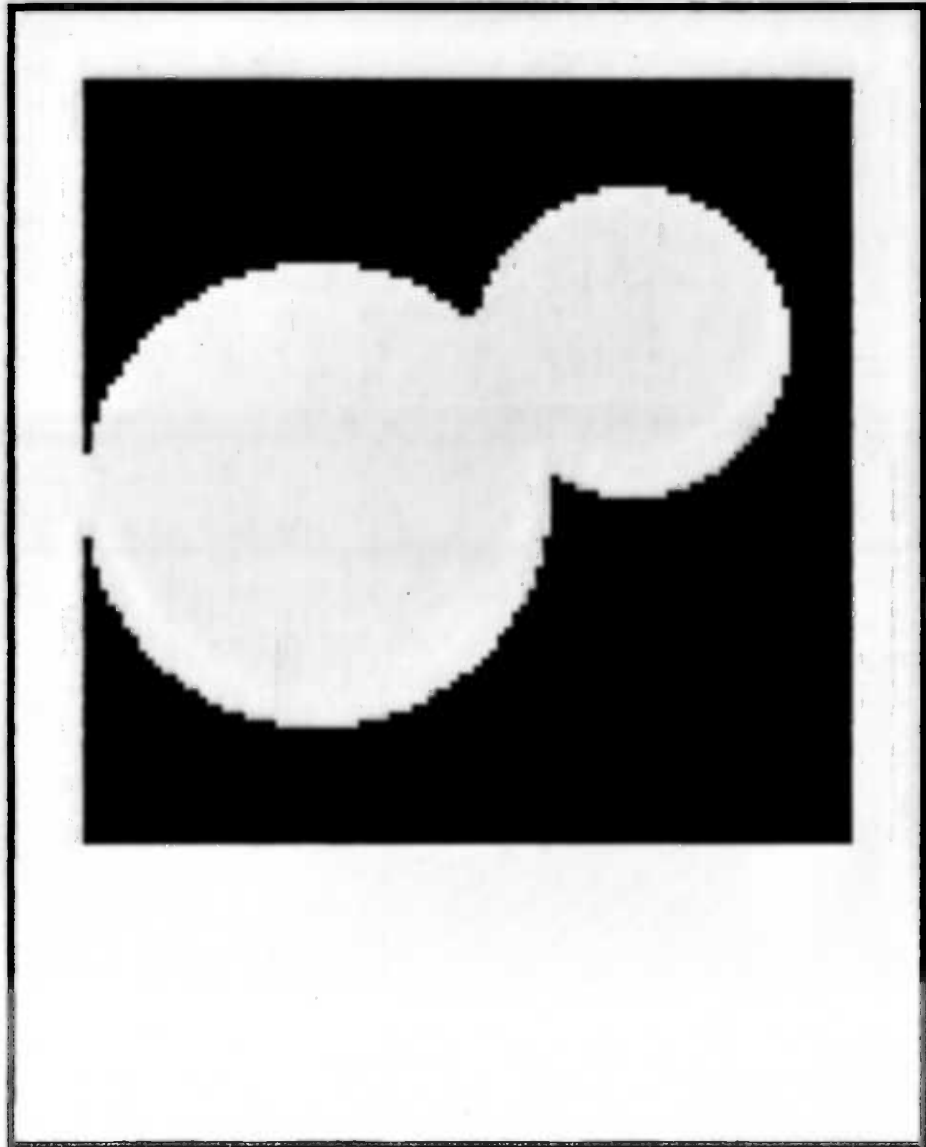


Figure 11. CIRC2 THRESHOLD 0-50.

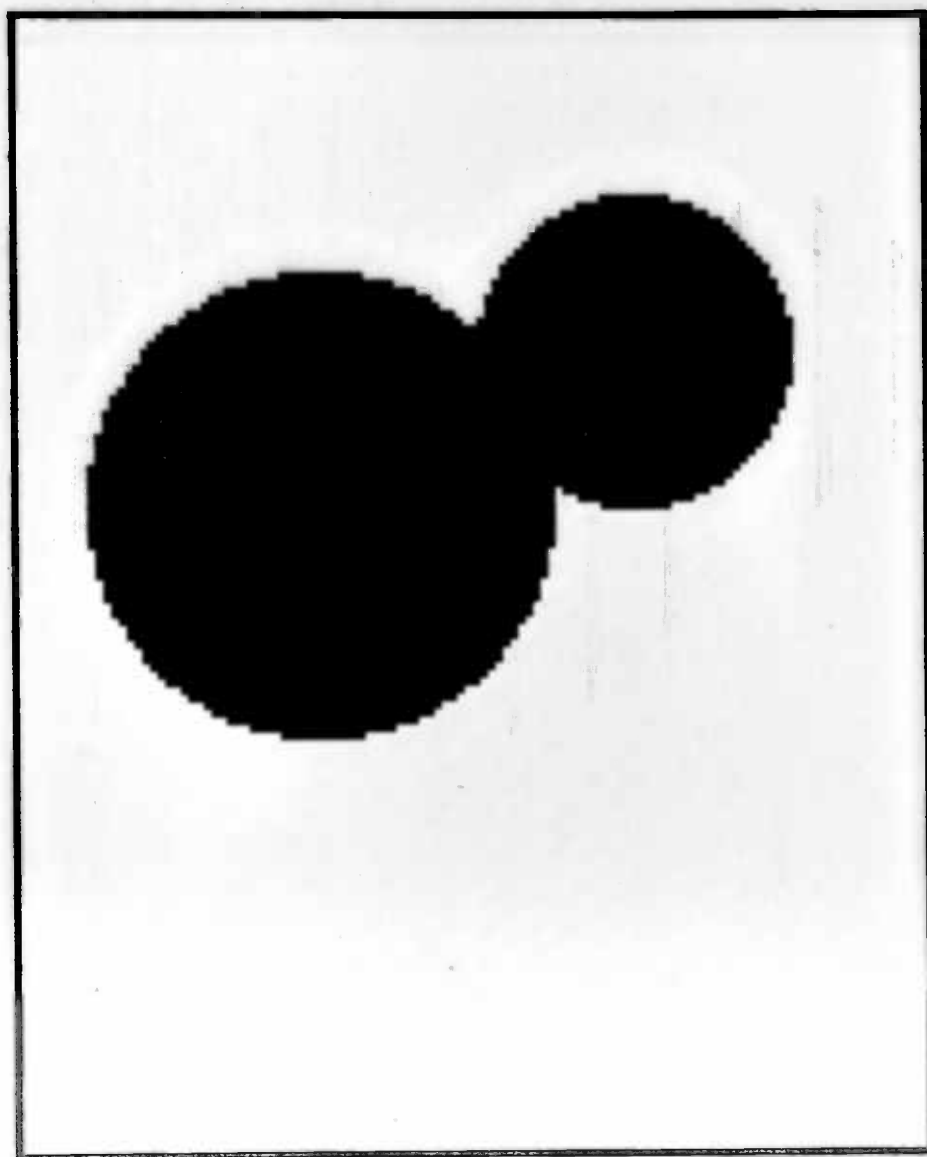


Figure 12. CIRC2 THRESHOLD 100-256.

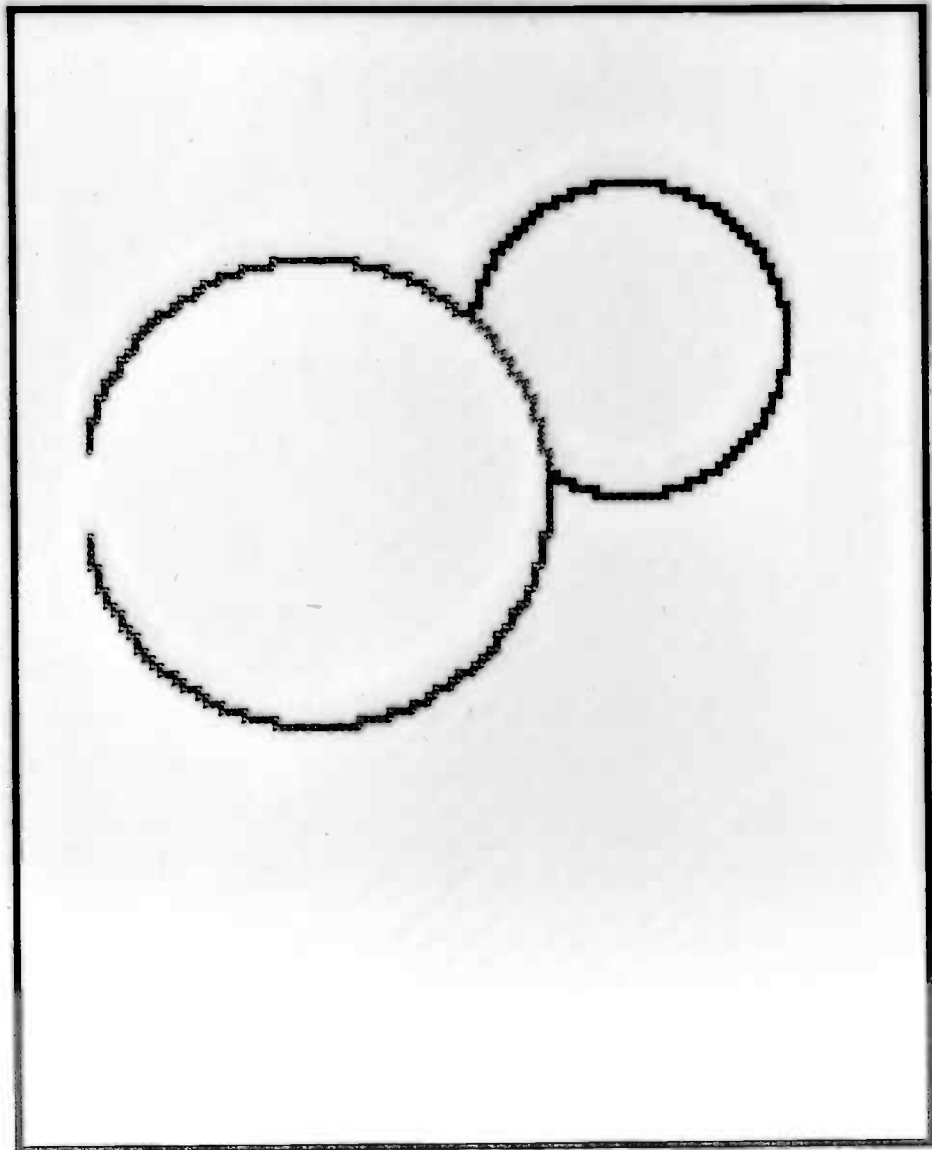


Figure 13. CIRC2 ROBERTS.

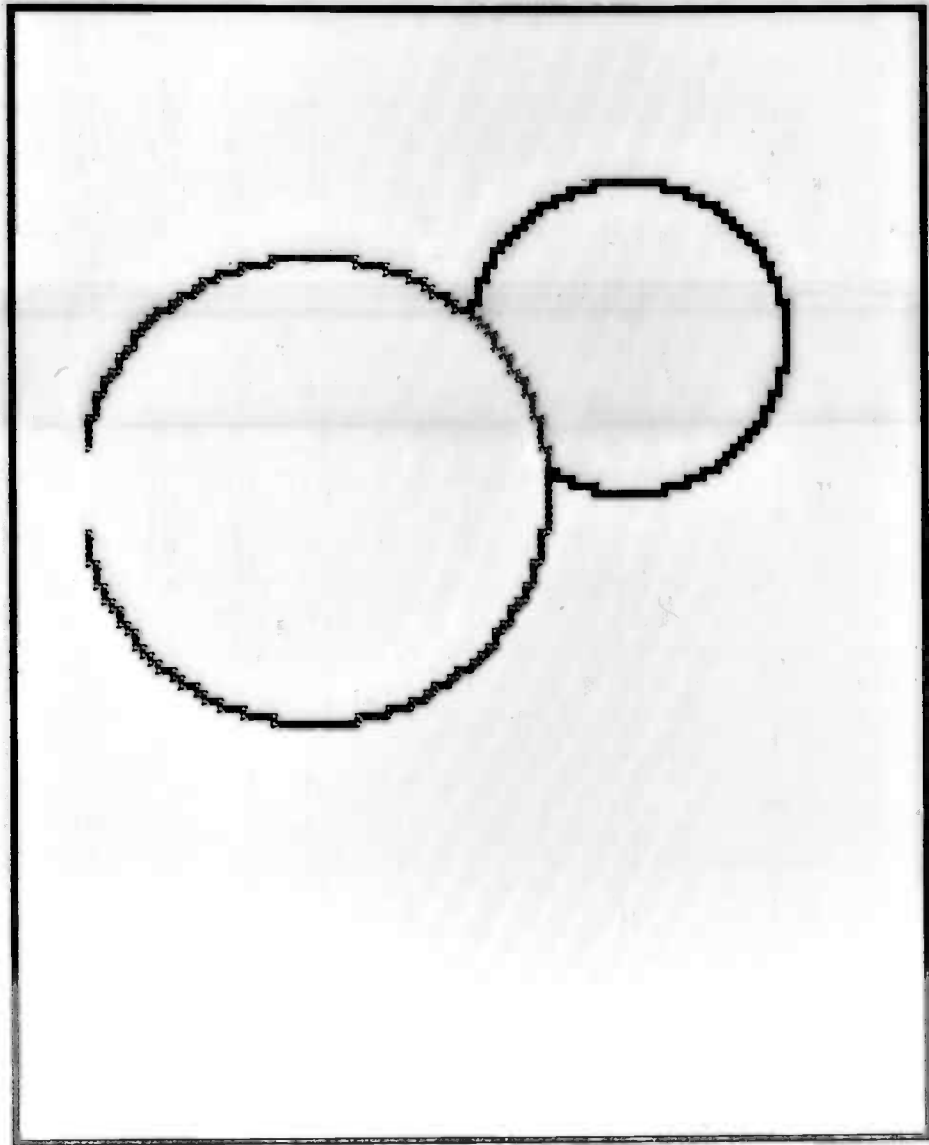


Figure 14. CIRCU ROBERTM.

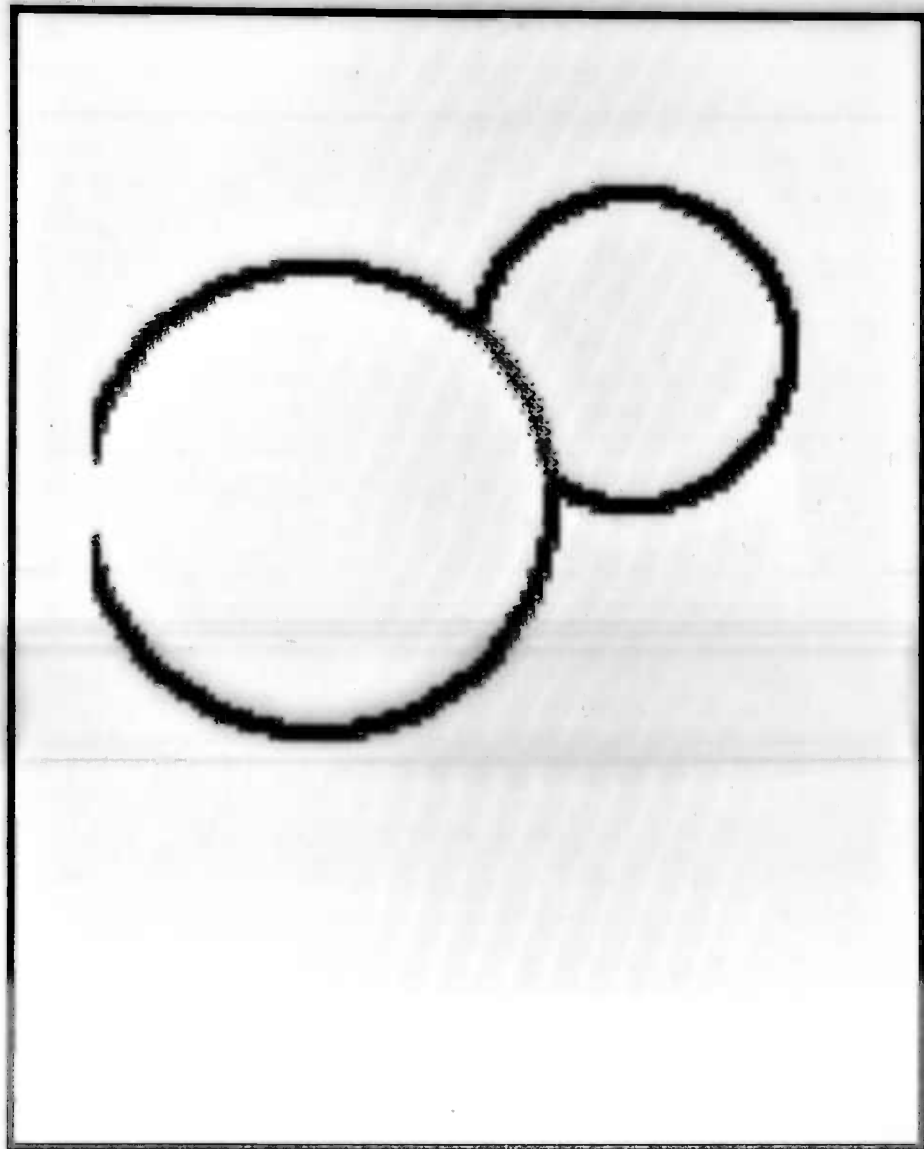


Figure 15. Circle SOBEL.

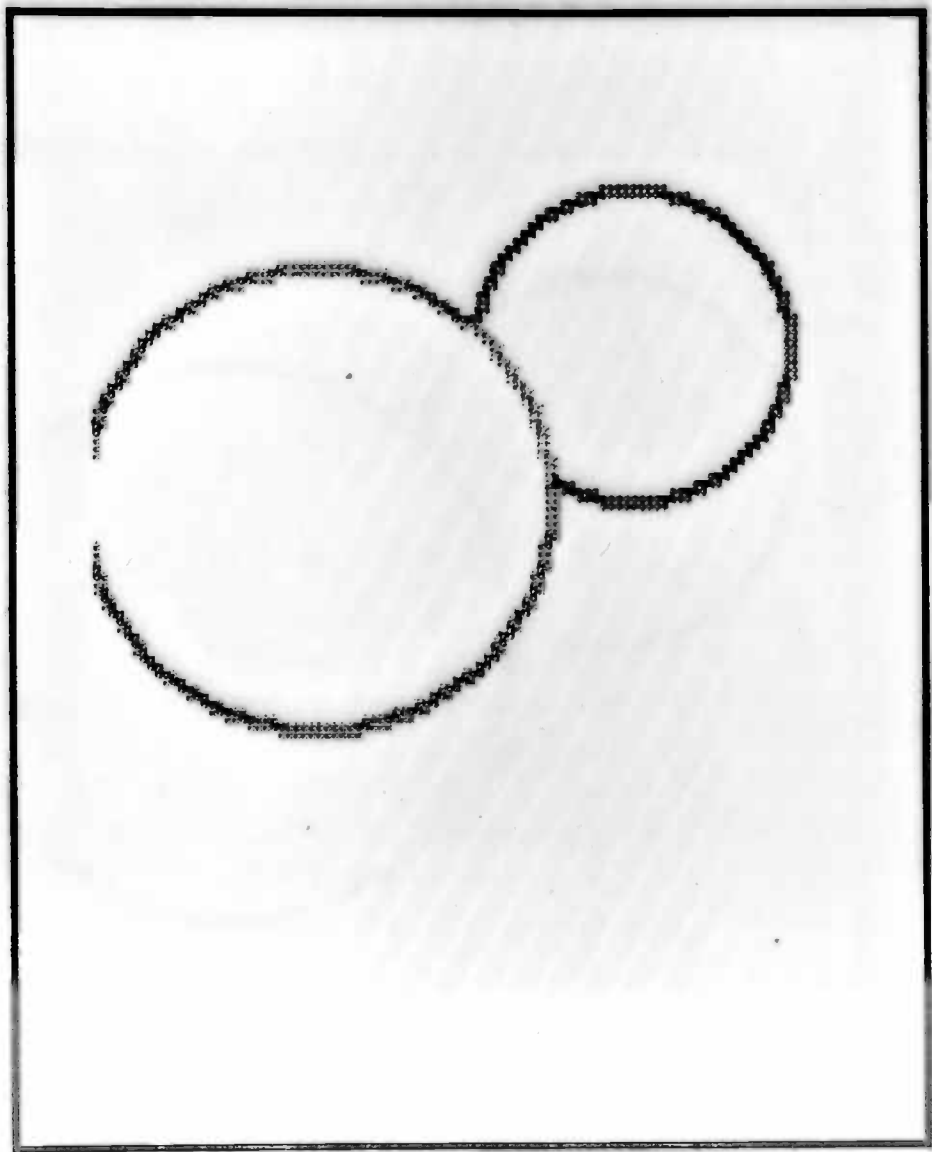


Figure 16. CIRC2 LAPLACE.

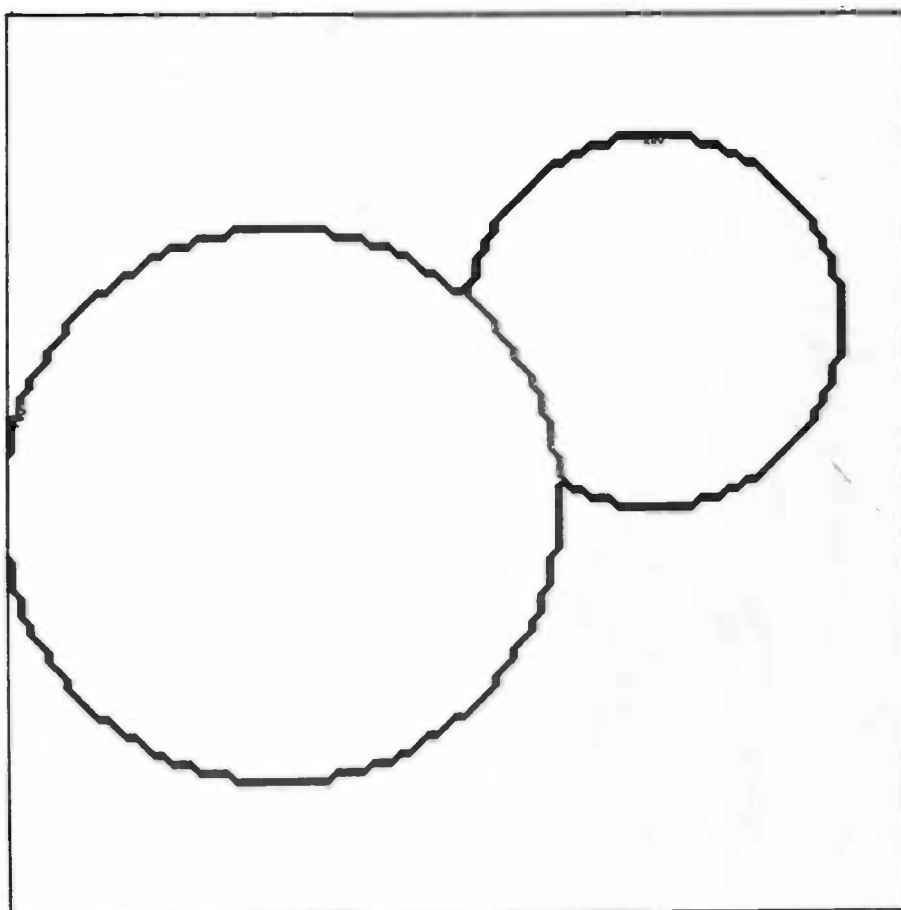


Figure 17. CIRC2 CONTOUR.

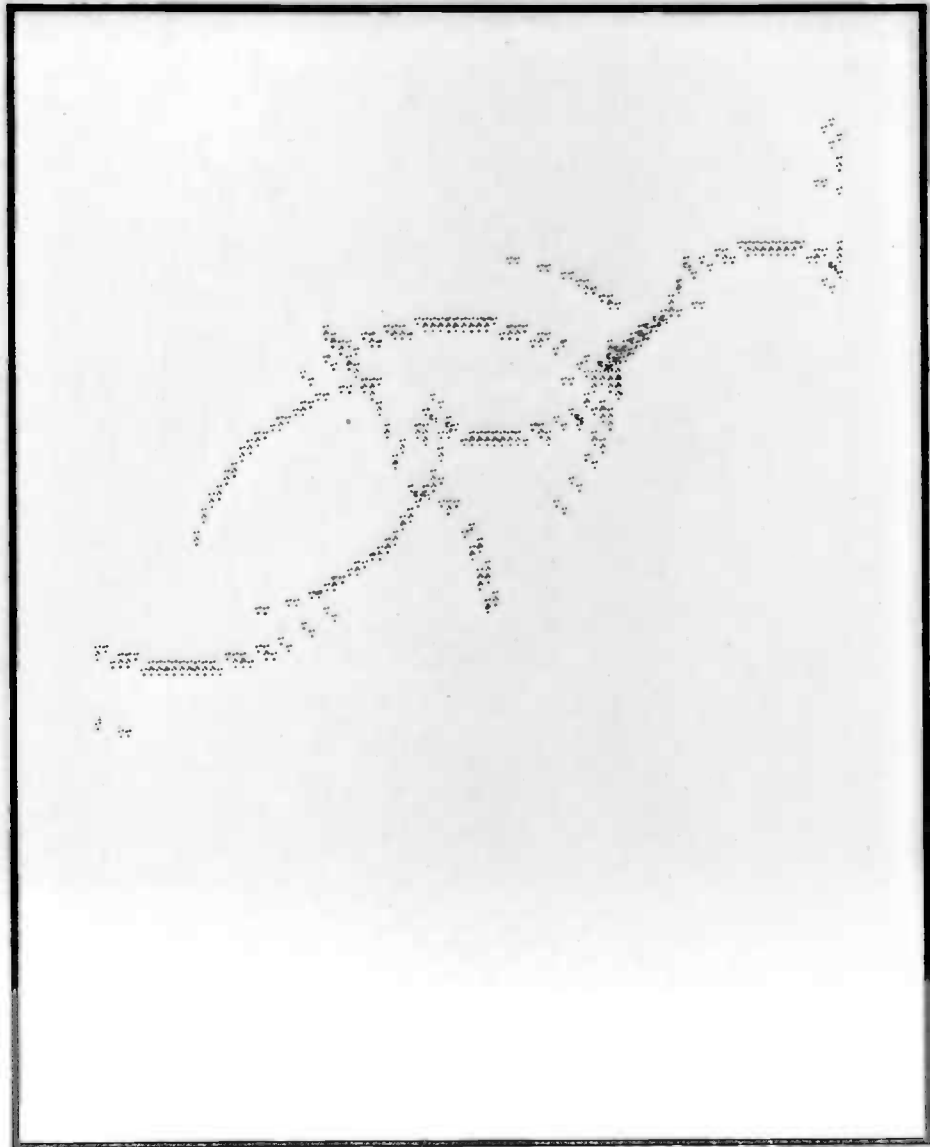


Figure 18. CIRC2 CIRCLES RADIUS 20.

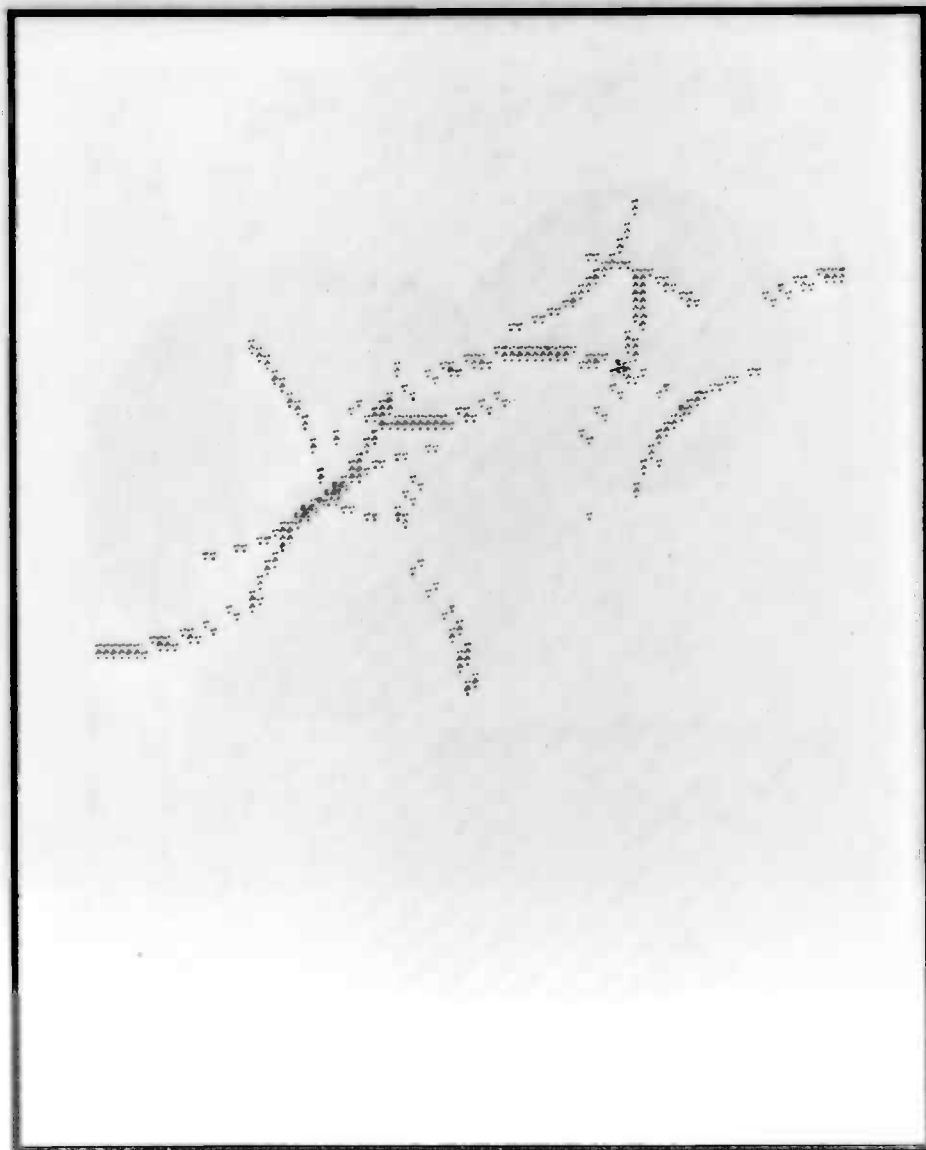


Figure 19. CIRC2 CIRCLES RADIUS 30.

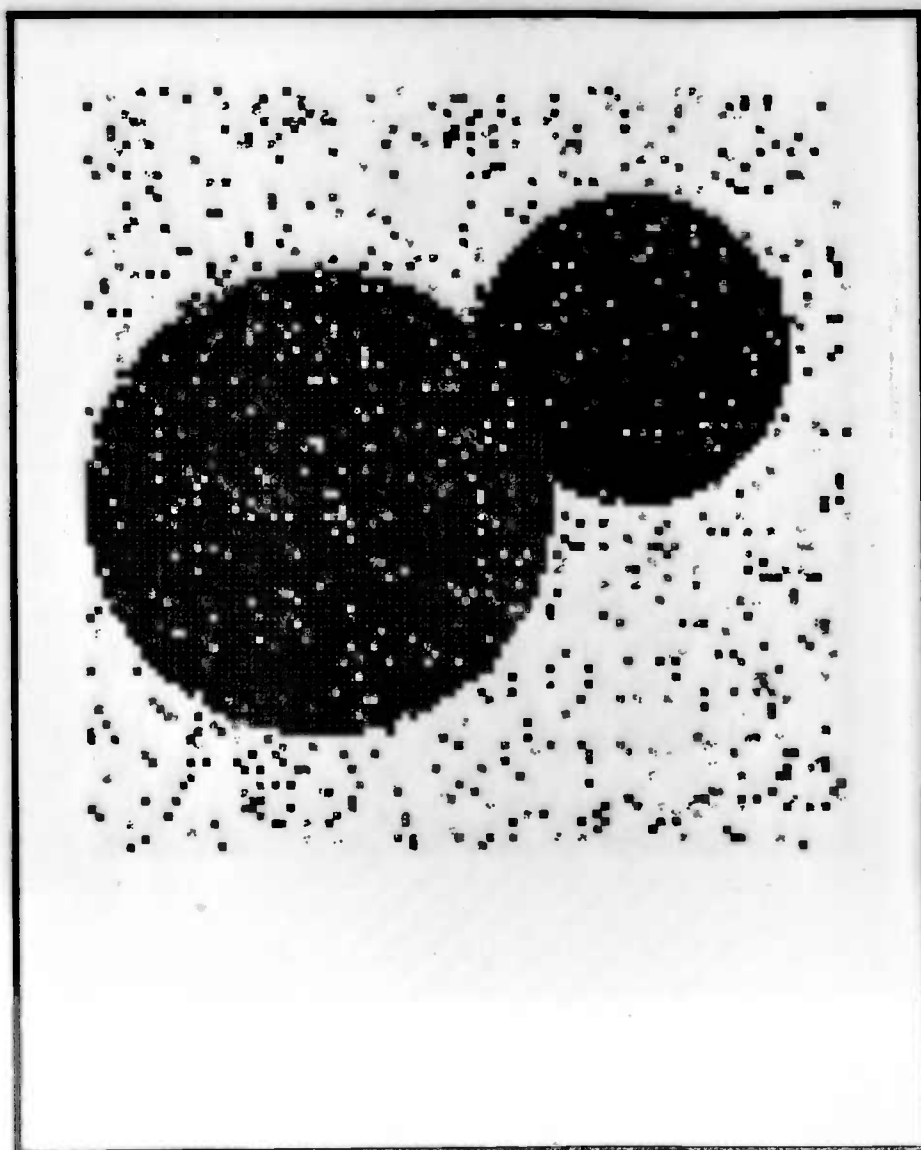


Figure 20. CIRC2N.



Figure 21. CIRC2N THRESHOLD 0-50.

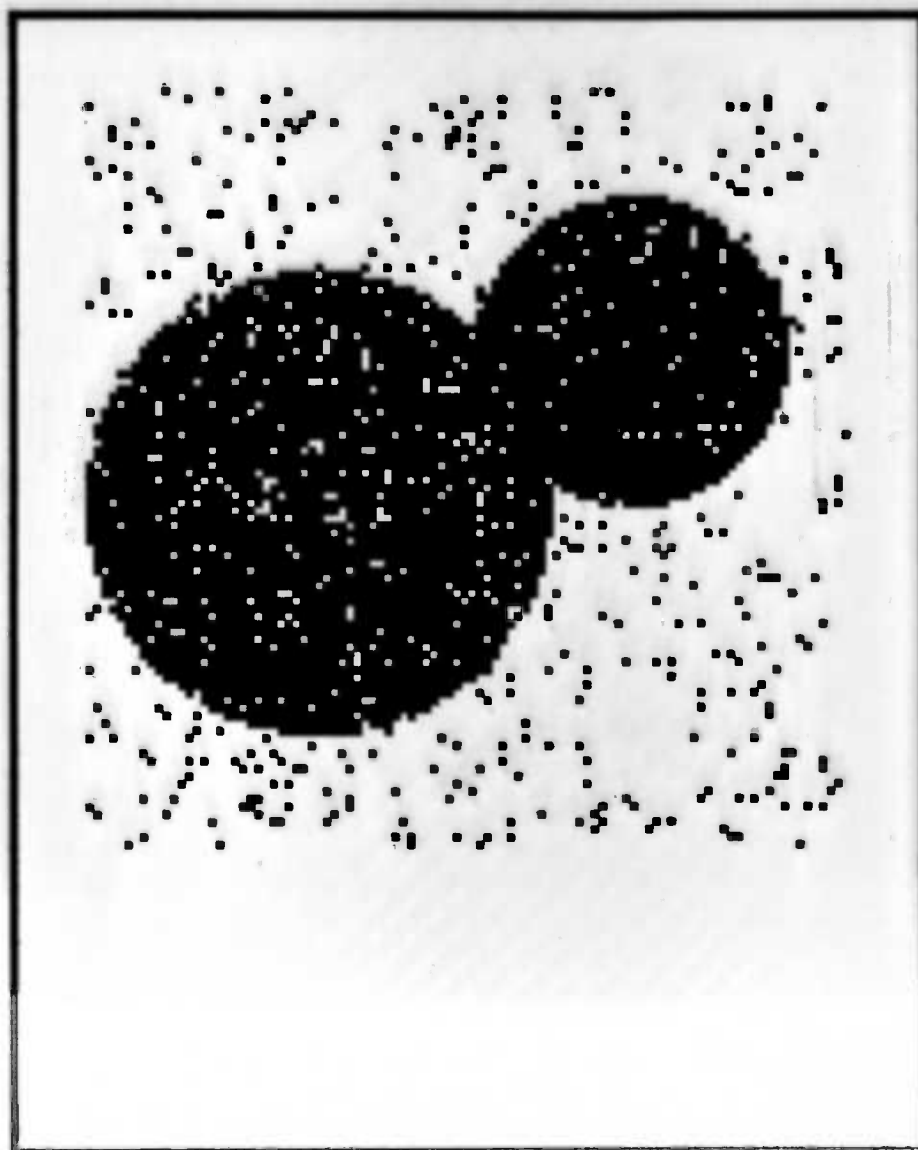


Figure 22. CIRC2N THRESHOLD 100-256.

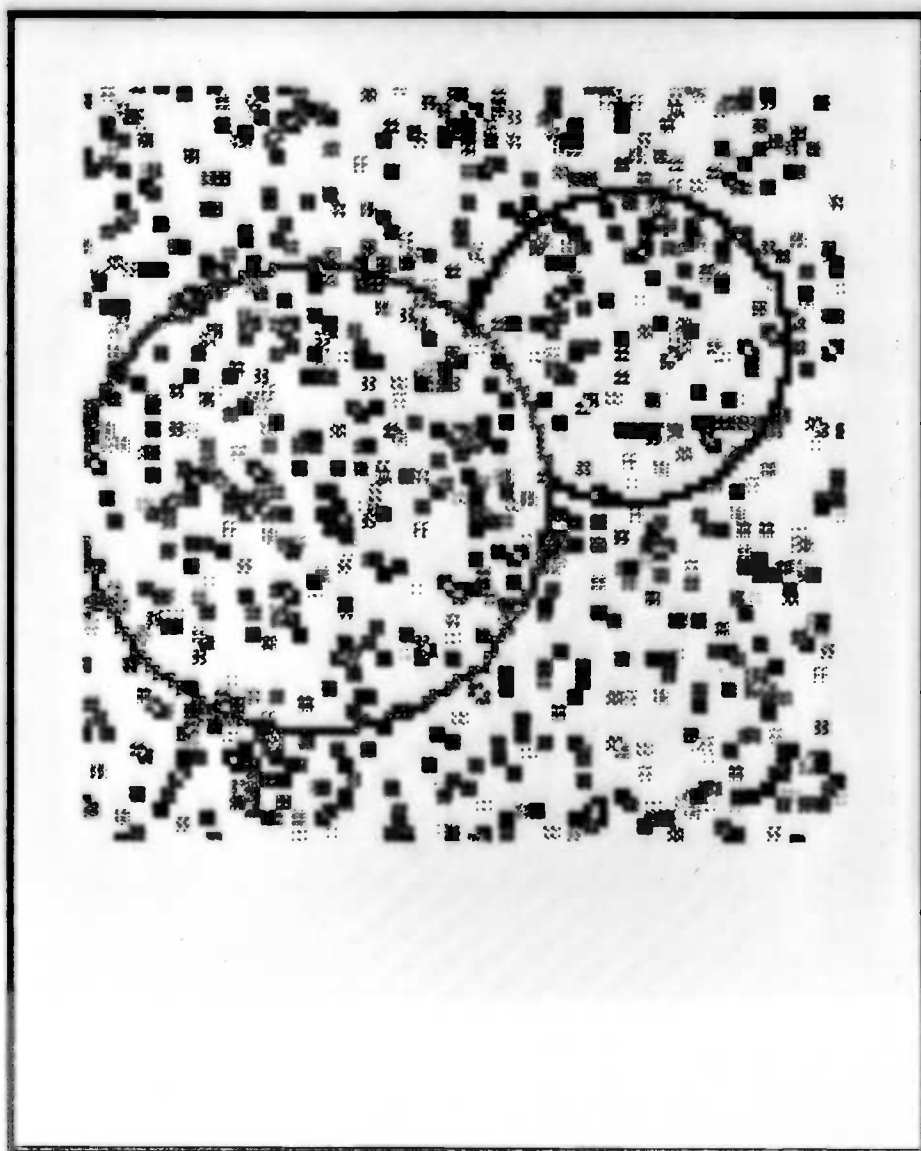


Figure 23. CIRC2N ROBERTS.

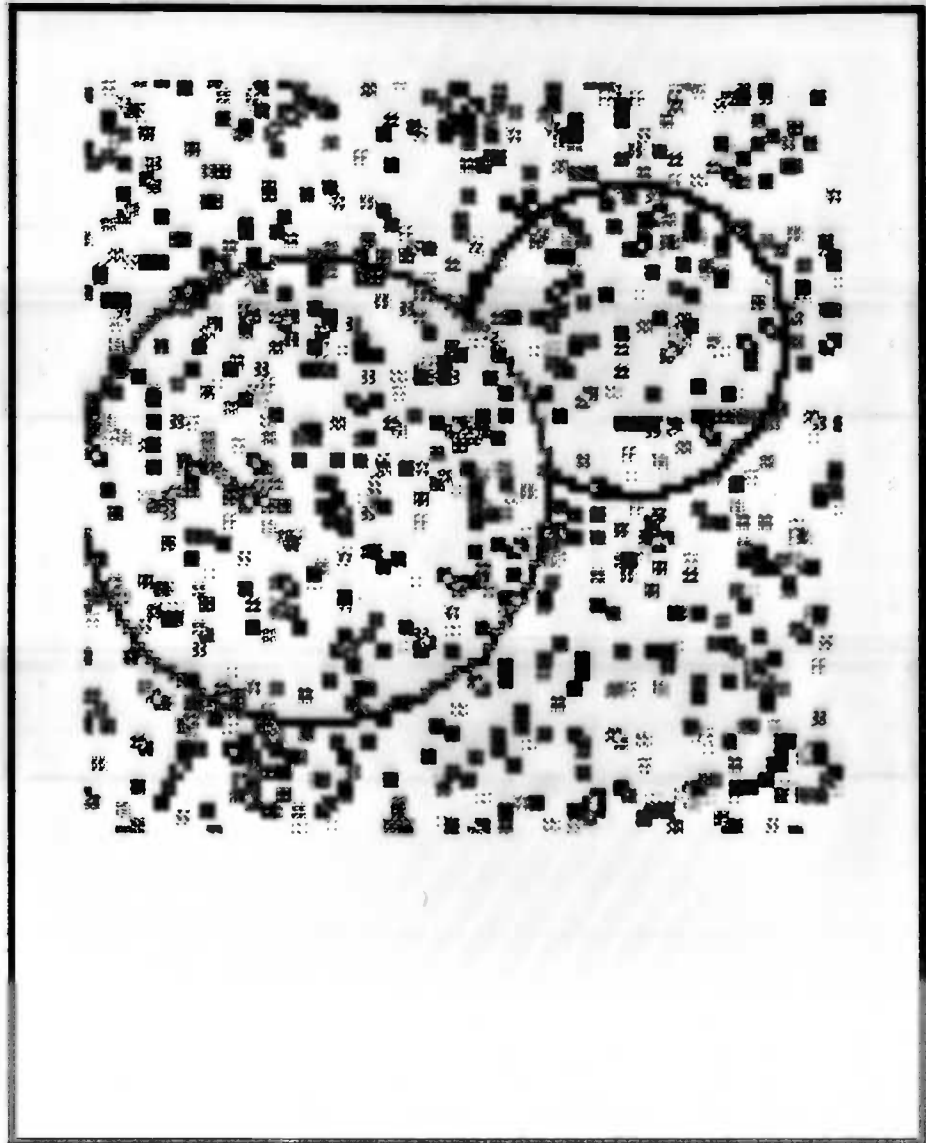


Figure 24. CIRC2N ROBERTM.

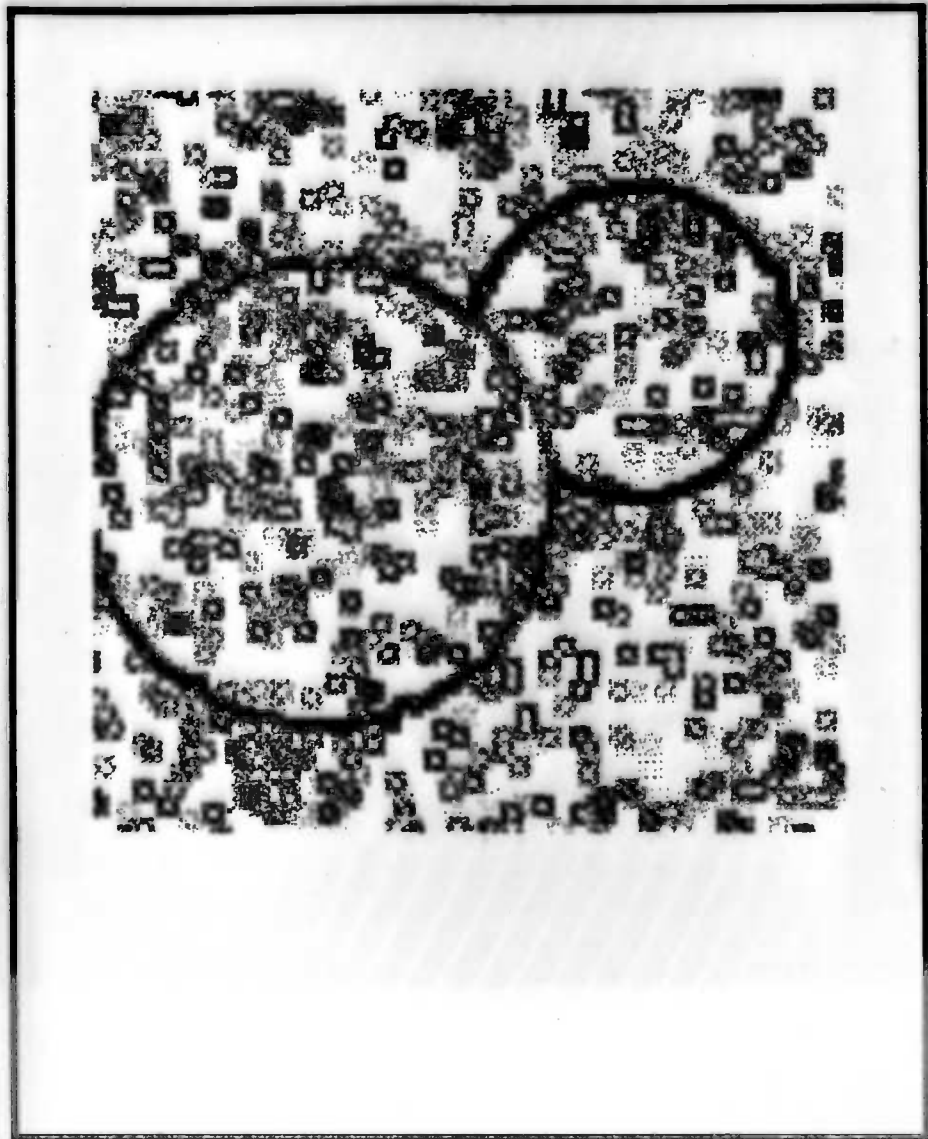


Figure 25. CIRC2N SOBEL.

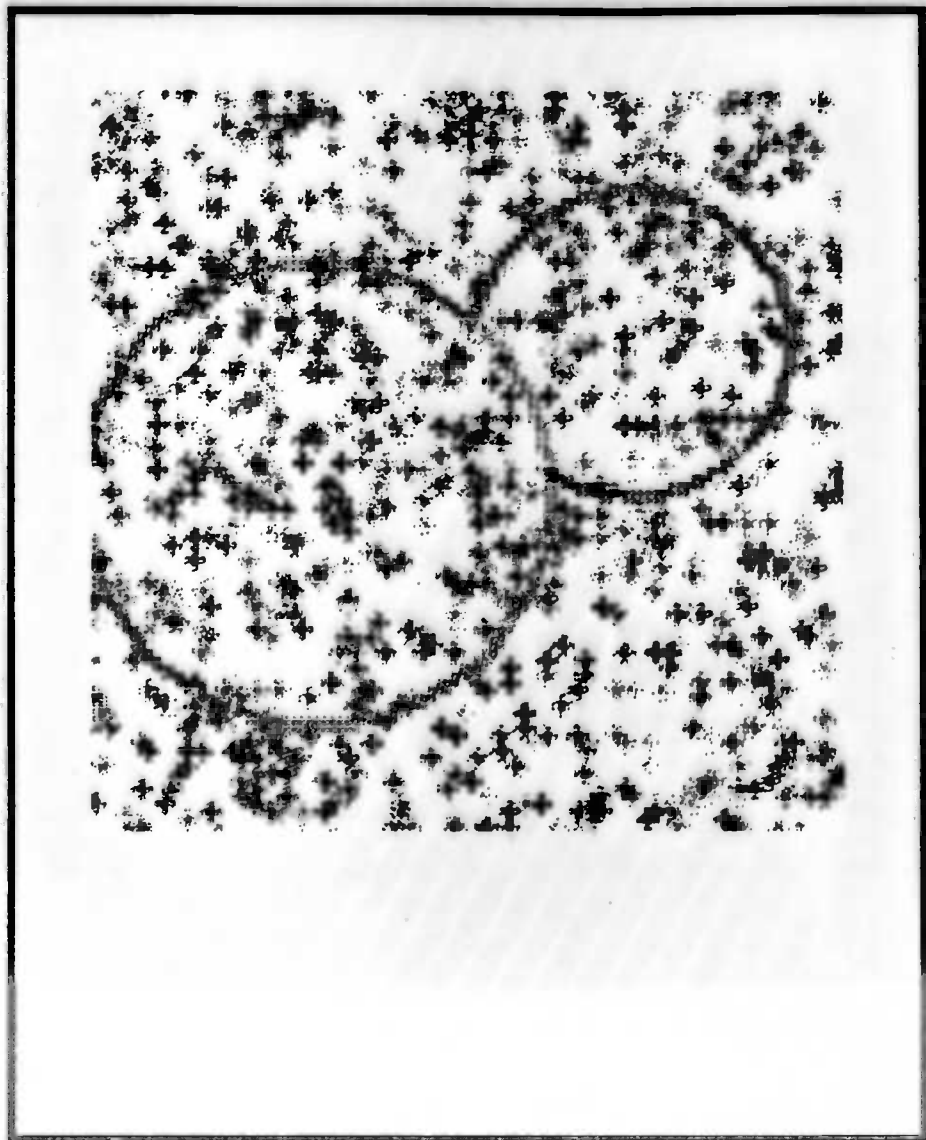


Figure 26. CIRC2N LAPLACE.

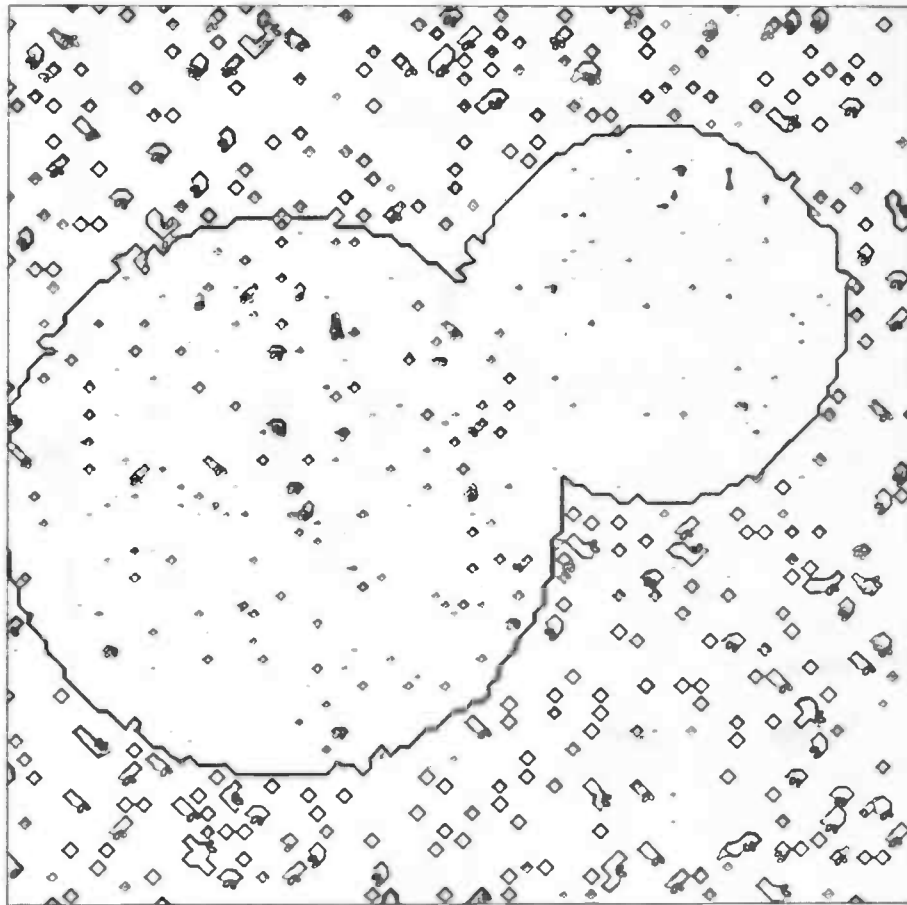


Figure 27. CIRC2N CONTOUR.

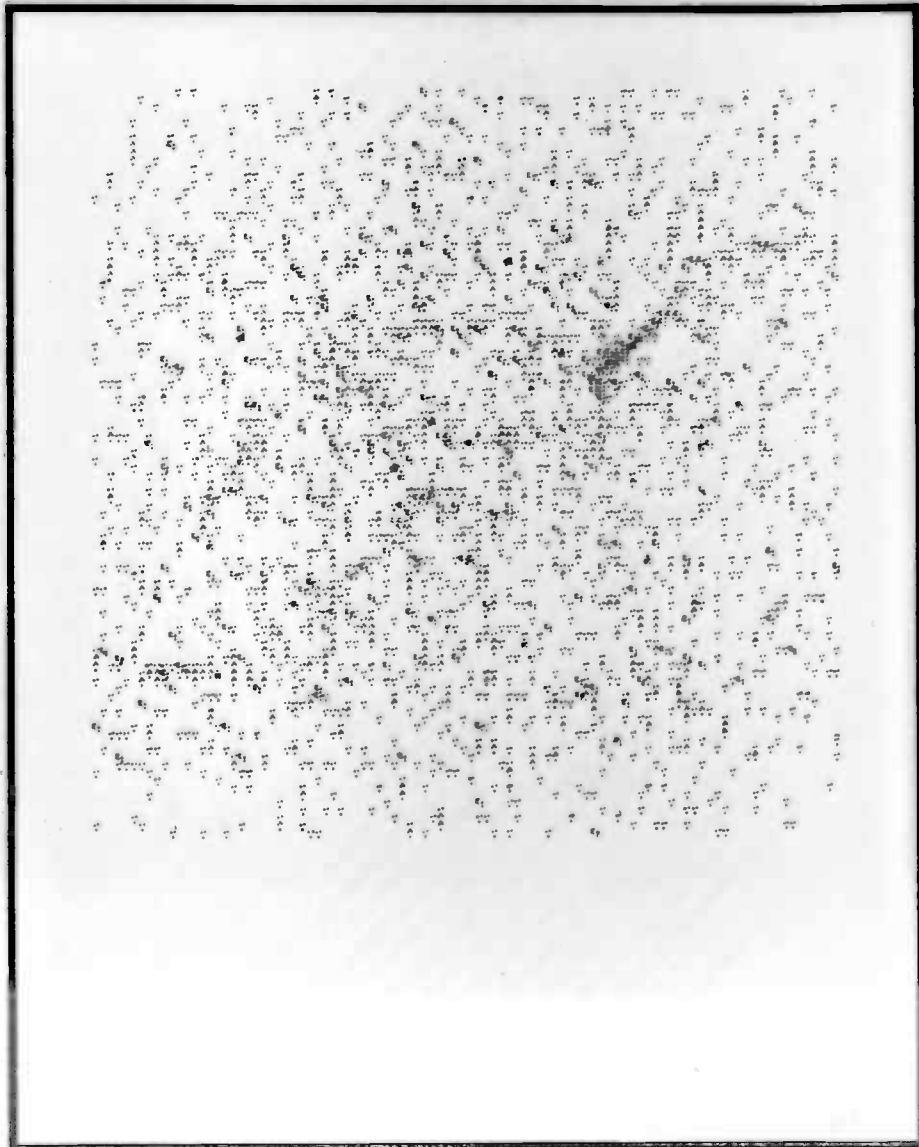


Figure 28. CIRC2N CIRCLES RADIUS 20.

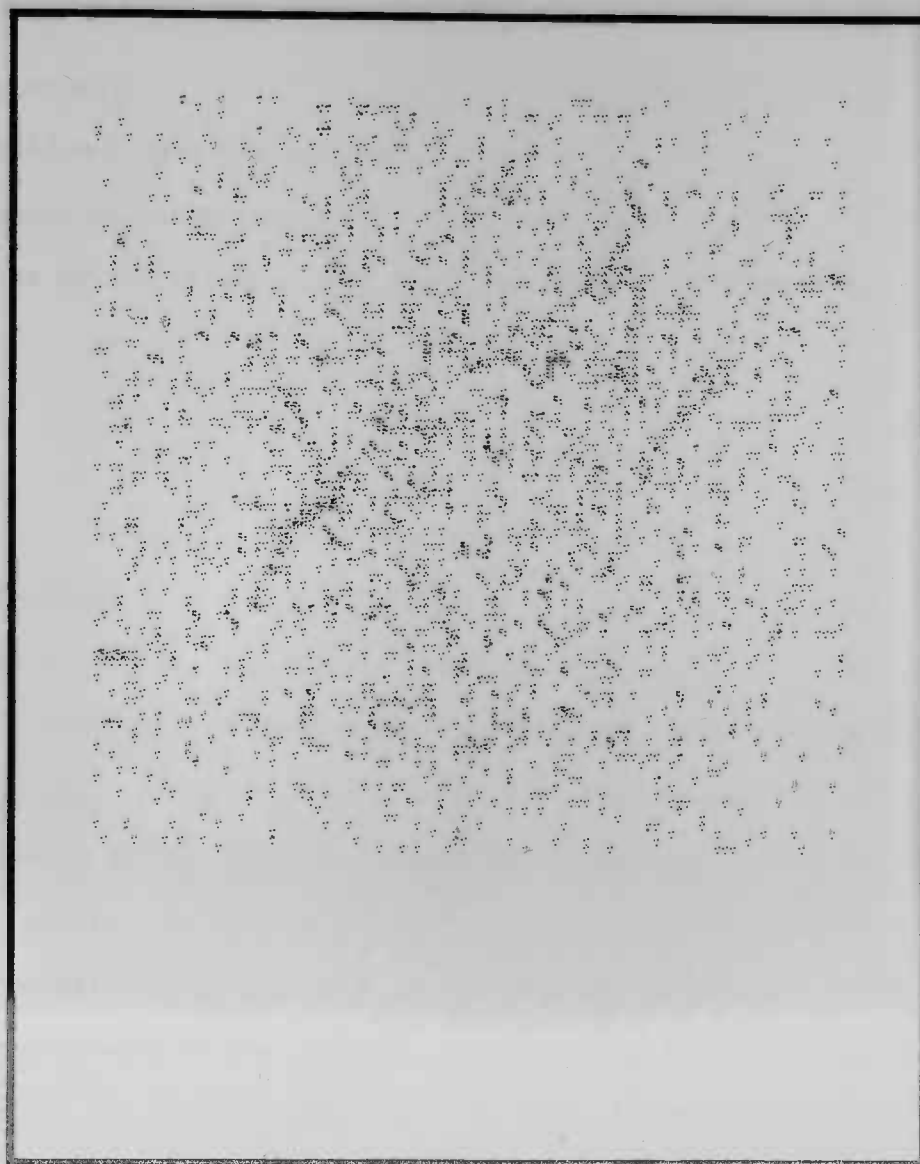


Figure 29. CIRC2N CIRCLES RADIUS 30.

5. EXAMPLE PROBLEM: THERMOGRAPHY.

5.1 Introduction.

5.2 The Problem-Orientated Approach.

5.3 Background to Thermography.

5.4 Examples of The Effect of Various Algorithms on Thermograms.

5.1 INTRODUCTION.

In this chapter, a particular problem is considered in detail, the early detection of breast cancer by analysing breast thermograms. This is really a description of the design of a particular image analysis system using the BLIPS package which was described in the previous chapter. In section 5.2, a description and justification of the problem-orientated approach to designing an image analysis system, as advocated by this thesis, is given. Section 5.3 gives some background to the use of thermography in breast cancer detection and specifically what the problem entails. Finally, section 5.4, a number of examples of the effects of different image analysis techniques on breast thermograms are described. As stated earlier the author feels that any developments in image analysis will come about through working with real data on real problems. This is why, along with implementing and developing the BLIPS package, a particular problem with realistic data has been studied. This chapter therefore covers a

particular application in detail.

5.2 THE PROBLEM-ORIENTATED APPROACH.

The package described in this thesis has been implemented to allow the development of specific image analysis systems for different problems and different types of data. This method, which I call 'the problem-orientated approach', facilitates the evaluation of suitable techniques for specific applications. The steps which an image analysis system designer goes through before investigating a new set of images are:

- 1) Start with some samples of the images to be analysed.
- 2) Consider whether the analysis of images such as these can be automated.
- 3) Decide how complex a system will be needed.

From considering the above vague questions to designing an actual image analysis system involves a series of refinements and formulations.

- 1) What can be inferred about the deterministic, probabilistic and mixed structure of the images?
- 2) What level of performance should the system strive for?
- 3) What competing design approaches are worth considering?
- 4) What manner of implementation is relevant?

The first step in the design of the system is to analyse the structure of the sample images to see which techniques could be applicable. To do this we need an interactive approach because:

- 1) It is now recognised that the key to image analysis problems does not lie wholly in learning machines, statistical approaches, spatial filtering, heuristic programming, formal

picture languages or any other particular solution which has been vigorously advocated by various groups during the last two decades as 'the solution to the image analysis problem'. No single model exists for all image analysis problems. Rather what we have is a 'bag of tools' and a 'bag of problems'.

- 2) Feature definition and extraction are best examined via trial and evaluation. What is needed is feedback between feature selection, logic design, classification and testing. This obviously points to the use of an interactive system.
- 3) Data analysis techniques try to answer questions about structure. Interactive analysis is a way of allowing examination of structure by 'humans' who are much better at making such decisions. To make the examination feasible we must provide suitable graphics for displaying these structural relationships.
- 4) The human must be the standard. We should put him in charge of the design and evaluation process but help him with automation in an interactive mode.

Quite simply we need a quick, flexible way of analysing sample images by trying out various 'tools' from a versatile 'tool kit' to determine which algorithm or approach should be selected for a given application. This exploratory approach is greatly enhanced by the automation of known procedures, so that they can be applied in a known fashion. This is what the BLIPS package was designed for and the remainder of this thesis pursues this approach in the design of a system for analysing breast thermograms.

5.3 BACKGROUND TO THERMOGRAPHY.

This section gives some background information on breast cancer, the use of thermography in breast cancer detection, thermographic scanners, breast cancer screening and the collection and storage of the data used in this thesis.

5.3.1 Breast Cancer.

Breast cancer is the most common cause of female cancer deaths. It is three times more common than any other female cancer and in addition it is about one hundred times more common in females than in males. Roughly one in five of all cancer deaths can be attributed to breast cancer, i.e. about twelve thousand deaths every year in the United Kingdom alone. The effect of early detection of the disease is to increase the five year survival rate. This is a figure which has been used to measure the effectiveness of various treatments of cancer, i.e. the number of people who survive for five years after cancer detection and treatment. It has been observed that breast cancer increases the surface temperature of the body in the location of the tumour thus making breast thermography a clinically useful tool in its detection.

5.3.2 Thermography.

Thermography is the pictorial representation of the thermal radiation from objects. Everything around us is generally at a temperature not far removed from 20 degree centigrade and is thus incandescent! This radiant glow, however, is in the infra-red part of the spectrum and is thus invisible. It is the conversion of this 'hot' scene into a visible one

which constitutes the thermographic technique. The technique has been with us for well over a decade and owes its development to the pressures of military requirements. Currently civil applications include aerial surveys of crops and river estuaries, assembly line location of electrical circuit faults, non-destructive testing, medical diagnosis of circulatory disorders, burns and cancer. A thermographic instrument consists basically of a system of rotating mirrors and prisms to scan the scene mechanically across an infra-red detector and produce a television type display. This rather inelegant technique is a feature of all scanners imposed on the designer by physical constraints. Objects at or near room temperature do not glow visibly but radiate well into the infra-red beyond the spectral limits of photographic films and television cameras.

The development of an inexpensive technique for automating the interpretation of breast thermograms by computer has important implications for the mass screening of breast cancer. The importance of screening for breast cancer has been frequently emphasised in the literature. Thermography is based on the physical principle that the amount of radiation emitted by an object depends upon its absolute temperature. The image obtained by an infra-red sensitive detector as it scans over a patient represents surface temperature variation. The observation that cancer, as well as other pathological mammary conditions, increases surface temperature resulted in the development of breast thermography as a clinical tool. Thermography is somewhat nonspecific but may be effectively used as a screening technique. Mammography, or breast xeroradiography which requires radiation exposure and considerable cost may then

be limited to patients with positive thermograms to further select those patients suitable for breast biopsy.

Many investigators do not make their criteria for thermographic diagnosis explicit. Absolute temperature has been regarded as having no diagnostic value. Symmetry and relative temperature are the basis from which most diagnostic criteria are developed. The ones described below are used in the Edinburgh Royal Infirmary where the Atomic Weapons Research Establishment MkII. scanner (OSB71) is used.

The criteria for an abnormal thermogram, using the Atomic Weapons Research Establishment MkII. scanner, are at present laid down as follows:

Areola temperature difference > 1.8 degree centigrade.

Average temperature difference > 1.0 degree centigrade.

'A' Factor > 0.044.

The average temperature calculation is based on the average of every temperature value in every fifth vertical line within the marked area of the breast. The reason for not using every column is that manually it is a very time consuming exercise.

The 'A' factor has been devised as follows:

$$\frac{\text{MAXIMUM TEMPERATURE(l)}}{\text{AVERAGE TEMPERATURE(r)}} - \frac{\text{MAXIMUM TEMPERATURE(r)}}{\text{AVERAGE TEMPERATURE(l)}}$$

Where (l) left shows the hottest temperature and vice versa for right.

It has also been pointed out that a further criterion is required for breasts showing a small localised area of increased heat emission. Some of these show up as abnormal by a positive

'A' factor but the smaller ones often do not. This has been under consideration and a preliminary attempt has been made to analyse these in more detail by, subdividing the total breast area into smaller squares and applying the full analysis to each of these squares individually, for comparison with the corresponding square on the other side.

Thermography can be considered a valid screening procedure despite the lack of perfect sensitivity and specificity if it maximises the yield of malignancies for a given expenditure. Many inaccuracies in thermographic interpretation are attributable to inconsistent application of poorly designed diagnostic criteria and to the limitations of the human visual system. Other shortcomings of manual interpretation for mass screening include cost and fatigue. If low cost automation of the thermographic examination is realised, then systematic screening of the entire population at risk becomes feasible for the first time.

5.3.3 Thermographic Scanner.

A thermographic scanner is an infra-red camera which measures the black body temperature in its field of view. Calibration stability is achieved with the aid of built in thermal references. Describing clinical thermographic data quantitatively in terms of black body temperatures is relevant, since the normal skin emissivity is very nearly unity and significant errors are introduced in oblique viewing of the surfaces only beyond 45 degrees.

On exposure, the skin takes some time to reach equilibrium with the environment and it is generally experienced that a

minimum of twenty minutes for a 20 degree centigrade ambient temperature is required. Surface temperatures on the face and body are then to be found in the 28 degree centigrade to 37 degree centigrade range, but the extremities can occasionally be lower. Signposts to the required thermal resolution for scanners are found in the work on breast cancer in which the average temperature difference between a malignant tumour and the contralateral site is 1.5 degree centigrade (LAW63), on burns where a full thickness burn results in a 3 degree centigrade drop (HAC70) and on carotid stenosis where forehead temperature variations are relevant (WOO64). A qualitative estimate of the required spatial resolution may be inferred from the cancer studies. With improved instrumentation 'hot' areas have frequently been found to be due to excessive vascularity with no obvious 'hot spot' spatially related to the tumour (DOD68). The 'hot spot' often turns out to be a vein junction in fact. The scanning instrument is calibrated in black body temperature terms and it should be noted that care is necessary in interpreting the data. When the black body temperature is close to the surface temperature, it is reasonable to use these terms and in practice a number of suitable subjects can be handled. Otherwise, the factors of low emissivity and oblique viewing must be taken into account.

Over the last decade thermography has become increasingly sophisticated especially in data presentation. Virtually all commercially available scanners now incorporate isotherm facilities either as bright line contours or multi-colour displays. Usually the built in thermal references are subject to ambient temperature variations and quantitative work has been

complex. The Atomic Weapons Research Establishment MkII. scanner has been constructed to facilitate quantitative medical thermography. A primary objective has been to produce accurate information in a numerically evaluated form. Stable thermal references have been included in the scanner to eliminate calibration and an unusually efficient optical system allows good spatial resolution and good signal to noise characteristics without sacrificing speed.

The obvious future improvements in thermographic scanners will come about through the addition of processing power and memory to the scanner, i.e. the addition of a computer or microcomputer. The Atomic Weapons Research Establishment have proposed the addition of a microprocessor to their MkII. scanner to provide a display of real-time and stored data. Also included in the proposals is an interactive facility to allow an operator to select a specific area of interest on the screen and parameters could then be calculated within the area, e.g. average temperature, maximum temperature, etc. A provision to store data for subsequent retrieval and analysis is included in the proposals.

5.3.4 Breast Cancer Screening.

A breast cancer screening program is currently underway in the United Kingdom. The aims of the project are to test the feasibility of a breast screening program and to act as a pilot scheme for the possible development of such clinics under the National Health Service. More specifically, in relation to this thesis, one aim is to evaluate thermography in terms of observer variation and false positive and false negative rates.

Unfortunately no permanent collection of the thermographic data, for subsequent analysis by computer, is being made.

The need for computer techniques for interpreting breast thermograms arises because of three problems.

- 1) The large amount of numerical data to be evaluated for each thermogram (see figure 30 showing a reproduction of a thermogram displaying the superimposed grid within which each grid cell is a temperature graded on a scale). As a result few if any thermograms are being fully analysed utilising this numerical and potentially informative data.
- 2) The large subjectivity and observer variation inherent in deciding upon the outline, or contour, of the breast in the thermogram and in the positioning of the areola, usually the area of minimum temperature.
- 3) The substantial subjectivity in the visual comparison of temperature patterns between the left and right breasts as is the current procedure and the difficulties in comparing thermograms taken at different times.

With the lessening of subjectivity and the collection of statistical parameters, it is hoped that it would become feasible to examine longitudinal changes in temperature patterns. The derivation of this information is, however, impossible without the prior and reliable delineation of the breast contour.

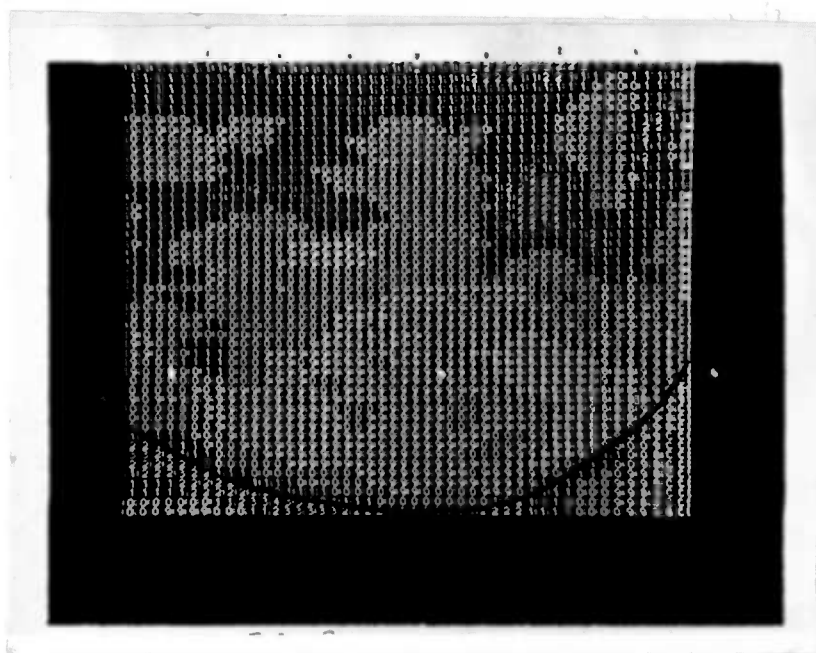


Figure 30. Numerical Grid.



Figure 31. Grey Scale Thermogram.

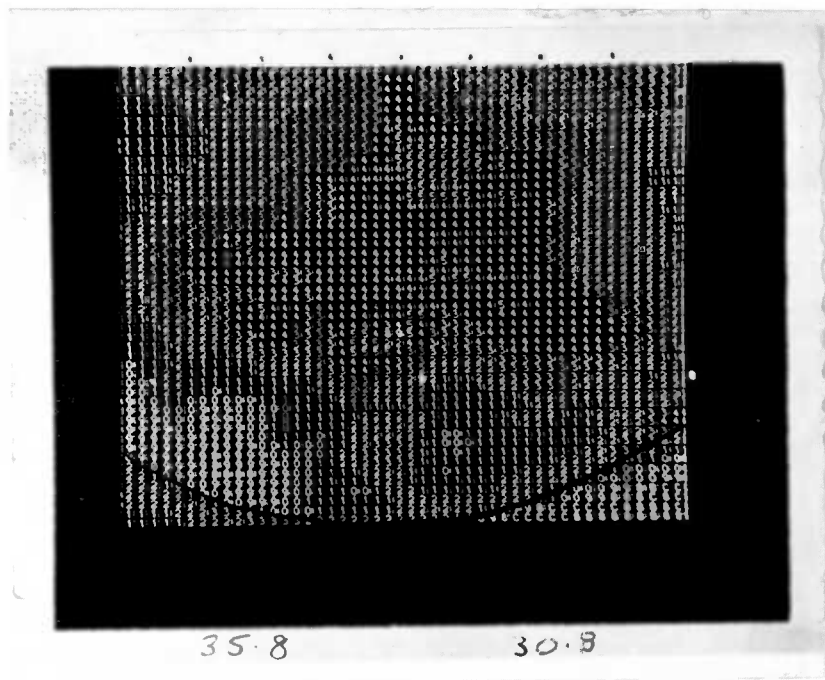


Figure 32. Numerical Grid.

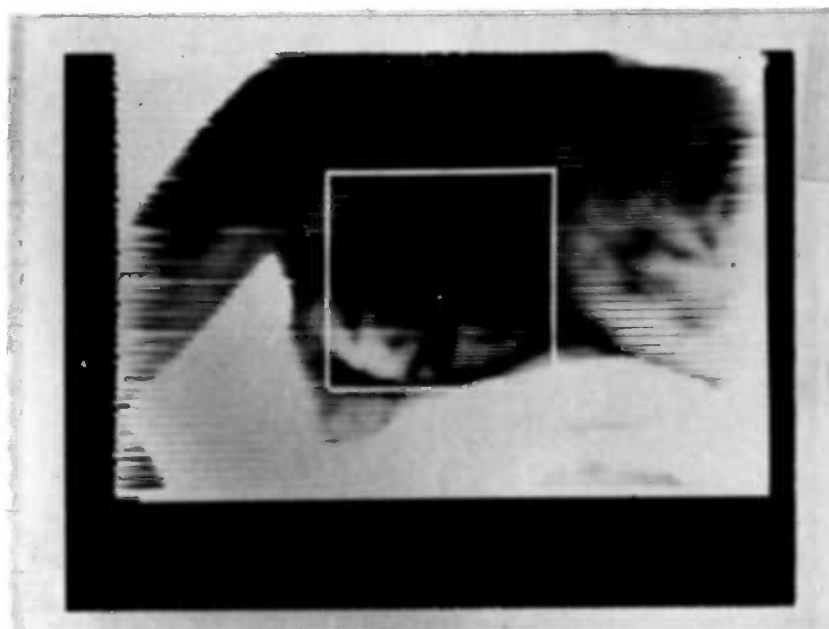


Figure 33. Grey Scale Thermogram.

5.3.5 Data Collection and Storage.

The seventy breast thermograms used in this research were taken in the Department of Diagnostic Radiology, University of Edinburgh between October 1973 and January 1974. The Atomic Weapons Research Establishment MkII thermographic scanner was used. Data collection suitable for computer input had not been attempted before this research was undertaken. The output from the thermographic scanner was two polaroid pictures (see figures 31 and 33 showing grey scale variation and figures 30 and 32 showing a cryptically encoded picture).

The data was collected from the Atomic Weapons Research Establishment MkII scanner before digitisation. Considerable trouble was encountered with the collection of the data as it was not originally recorded with a view to analysis. In fact the data was collected on an analogue tape recorder purely to facilitate the maintenance and checking of the scanner. The lack of suitable equipment for digitisation also resulted in the data being digitised by running the tape through the analogue tape recorder at a quarter of the speed at which it was originally recorded to get a suitable bandwidth, estimated at 16 kilohertz, and sampling the data at a rate of 3750 points per second, i.e. giving an effective bandwidth of 15 kilohertz. The data was then logged to magnetic tape and each image in the data which had been digitised had to be identified manually and subsequently extracted into a file. Each line of the thermogram had to be identified and the data was then stored in a suitable format. It was possible to write a program to chop the thermogram up into separate lines although the lines varied in length from 113 to

115 points. The short lines were interpolated to full size for consistency. Each image was then 100 lines with each line 115 points in length, i.e. 11500 data points per image. There was unfortunately no scaling information available with the data to associate it with actual temperatures. Also there was no way to identify each thermogram with the individual patients which they were taken from. All of the data collected from the thermographic scanner, seventy images in all, was eventually stored in a partitioned data file on EMAS. Each image was then easily accessible and could be manipulated by name, e.g. PICFILE_THERM33. Each image requires 23 kilobytes when stored in the file, i.e. 2 bytes/pixel, but requires 50 kilobytes when being analysed, i.e. 4 bytes/pixel. This is because the images are held in integer arrays to allow routines from standard packages to manipulate them.

5.4 EXAMPLES OF THE EFFECT OF VARIOUS ALGORITHMS ON THERMOGRAMS.

In this section some examples of the effect of various algorithms on thermograms are given. Three thermograms have been chosen from a selection of seventy to demonstrate the result of applying various algorithms. No evaluation of the different techniques is made in this section that is fully discussed in chapter 6. The three thermograms used are, a view concentrating on the right breast, a central view and a view concentrating on the left breast. They have all been taken from the same patient, this was ascertained from an identifiable necklace or pendant being present in each image.

The algorithms which were applied are very similar to those applied to the phantoms described in chapter 4. To simplify the

correlation of the techniques with the figures shown at the end of this chapter the application of the various techniques using the BLIPS package is reproduced below along with a commentary.

The three images used are referred to as THERM58, THERM59 and THERM60. They were scaled during the course of the analysis and saved to files. These new scaled images are referred to as THERM58S, THERM59S and THERM60S. All six images were stored in the partitioned data file THERMS. Each technique is applied to each thermogram in turn to make their comparison simple.

Analysing The Thermograms.

BLIPS(THERMS)	Call the BLIPS package with the partitioned file THERMS as a parameter.
GET(THERM58)	Get THERM58 and find the
RANGE	maximum and minimum values within it.
MAX 183 at line 69 col 19	The package prints the range
MIN -129 at line 1 col 52	
NEGATE(-129,183)	Negate the image to make the low values high and the high values low.
SCALE(-129,100)	Scale the image making the value -129 equal to 0. Leave the amplitude constant.
SHADE(0,312)	Shade the resultant image.
	Reproduced in figure 34.

GET(THERM59)

The same operations are carried out on THERM59.

RANGE

MAX 195 at line 62 col 25

MIN -128 at line 76 col 54

NEGATE(-128,195)

SCALE(-128,100)

SHADE(0,323)

Reproduced in figure 35.

GET(THERM60)

The same operations are carried out on THERM60.

RANGE

MAX 198 at line 74 col 70

MIN -134 at line 24 col 25

NEGATE(-134,198)

SCALE(-134,100)

SHADE(0,332)

Reproduced in figure 36.

GET(THERM58)

Get THERM58 into the current image and scale it to a consistent

NEGATE(-130,50)

range. Threshold out the background.

SCALE(-130,100)

SAVE(THERM58S)

Save the resulting image to file THERM58S.

SHADE(0,180)

Reproduced in figure 37.

GET(THERM59)

The same operations are carried out on THERM59.

NEGATE(-130,50)

SCALE(-130,100)

SAVE(THERM59S)

SHADE(0,130)

Reproduced in figure 38.

GET(THERM60)	The same operations are carried
NEGATE(-130,50)	out on THERM60.
SCALE(-130,100)	
SAVE(THERM60S)	
SHADE(0,180)	Reproduced in figure 39.

The following operations are carried out on all three thermograms. The use of the abbreviation THERM will represent as appropriate THERM58S, THERM59S and THERM60S. The three images produced by each algorithm are reproduced at the end of this chapter.

GET(THERM)	Get the appropriate thermogram into the current image.
ROBERTS	Apply the ROBERTS operator.
SHADE(0,175)	Figures 40,41,42
THRESHOLD(85,175)	Threshold the result at 50% of the maximum value.
SHADE(0,1)	Figures 43,44,45
THRESHOLD(45,175)	Threshold the result at 25% of the maximum value.
SHADE(0,1)	Figures 46,47,48
GET(THERM)	Get the appropriate thermogram into the current image.
ROBERTM	Apply the Robertm operator.
SHADE(0,250)	Figures 49,50,51
THRESHOLD(125,250)	Threshold at 50%.
SHADE(0,1)	Figures 52,53,54
THRESHOLD(65,250)	Threshold at 25%.
SHADE(0,1)	Figures 55,56,57

GET(THERM)	Get the appropriate thermogram.
SOBEL	Apply the Sobel operator.
SHADE(0,550)	Figures 58,59,60
THRESHOLD(275,550)	Threshold at 50%.
SHADE(0,1)	Figures 61,62,63
THRESHOLD(135,550)	Threshold at 25%.
SHADE(0,1)	Figures 64,65,66
GET(THERM)	Get the appropriate thermogram.
LAPLACE	Apply the Laplace operator.
SHADE(0,275)	Figures 67,68,69
THRESHOLD(135,275)	Threshold at 50%.
SHADE(0,1)	Figures 70,71,72
THRESHOLD(65,275)	Threshold at 25%.
SHADE(0,1)	Figures 73,74,75
GET(THERM)	Get the appropriate thermogram.
CONTOUR(0,10,180)	Display the contour levels from 0 to 180 in increments of 10. Figures 76,77,78

The images resulting from the algorithms are reproduced here. The following chapter evaluates the different techniques useful in the analysis of breast thermograms. Only a small sample of a set produced by the author (LAI77) have been described; it would not be feasible to reproduce any more images in this thesis.

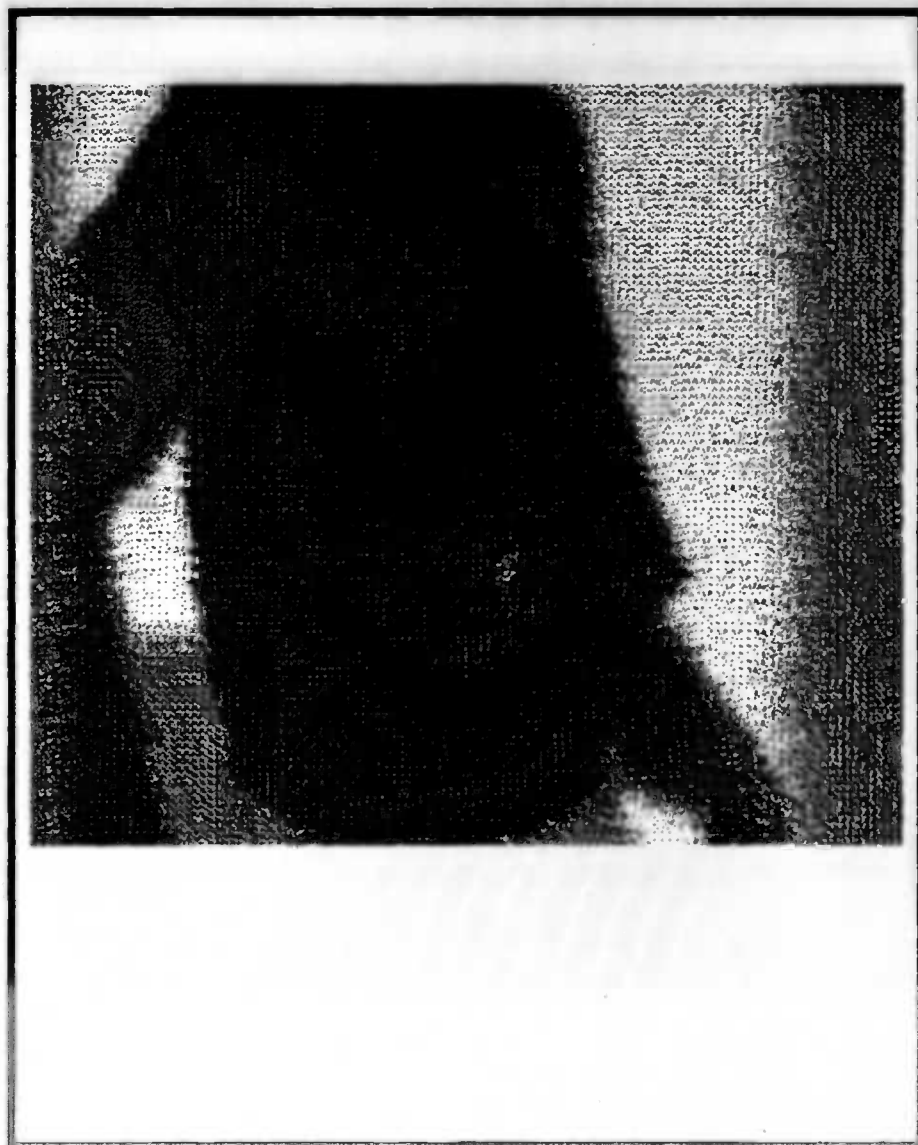


Figure 34. THERM58.

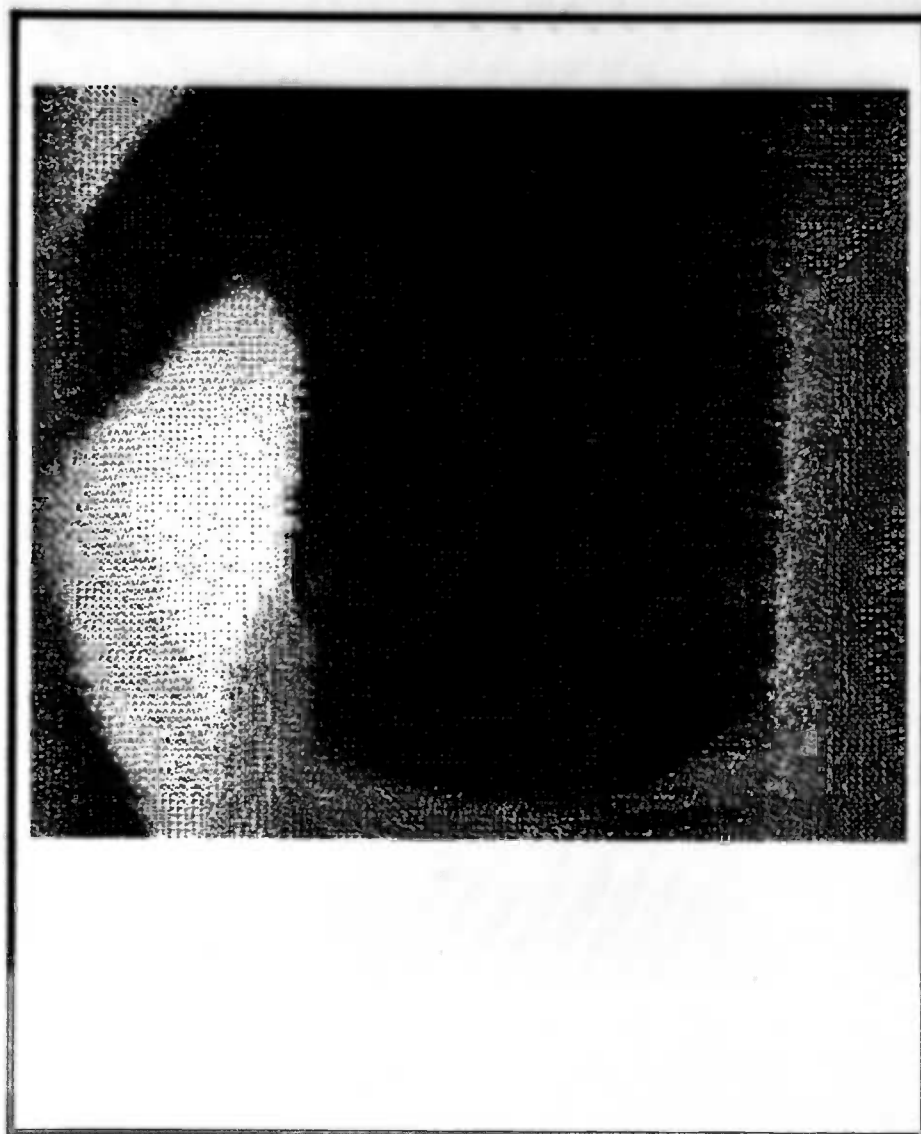


Figure 35. THERM59.



Figure 36. THERM60.



Figure 37. THERM58S.



Figure 38. THERM595

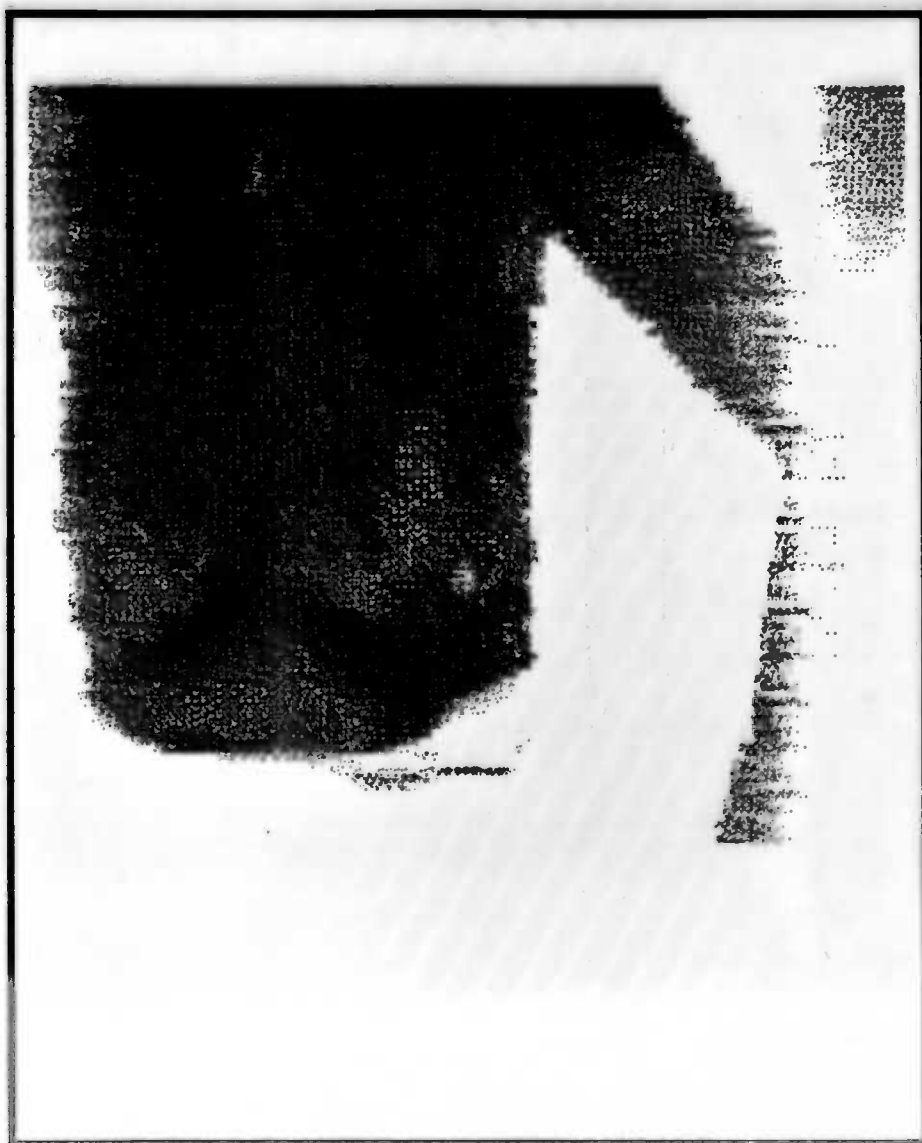


Figure 39. THERM60S.

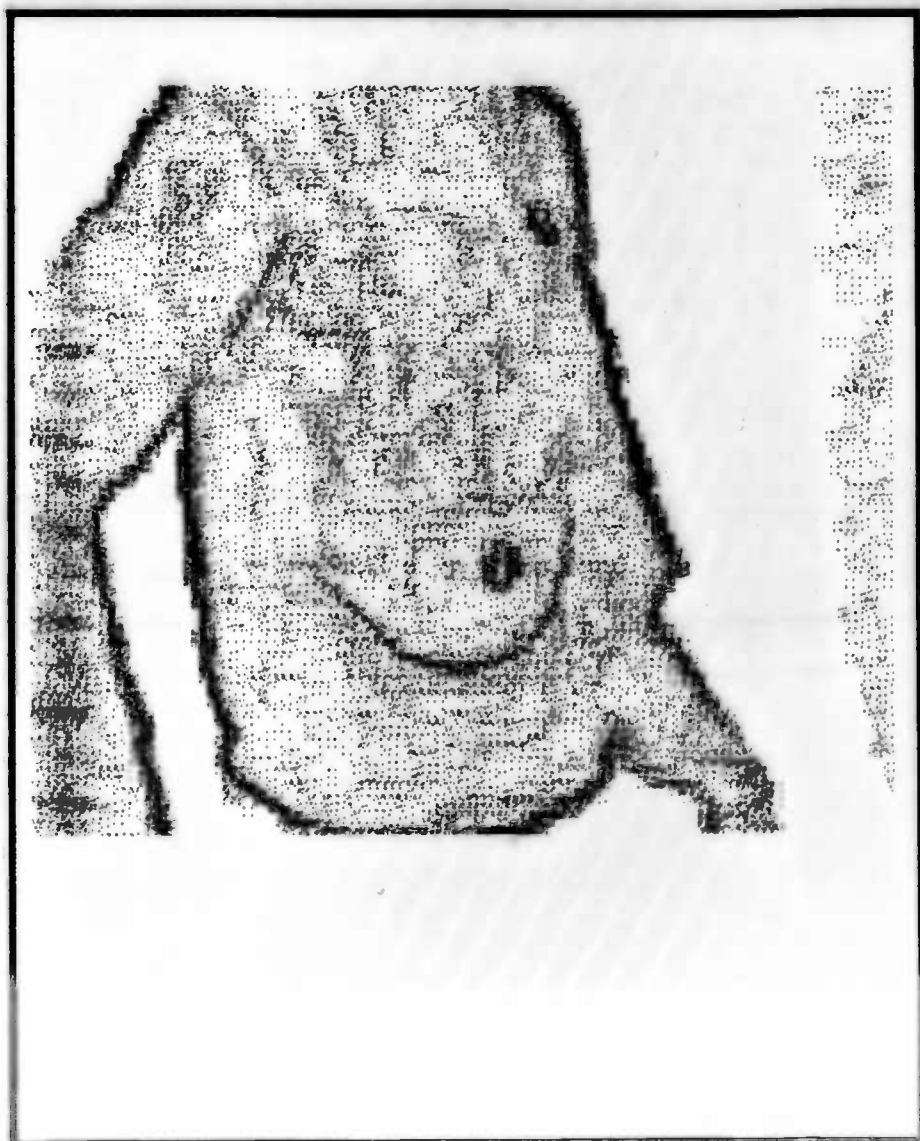


Figure 40. THERMS8S ROBERTS.



Figure 41. THERMS9S ROBERTS.

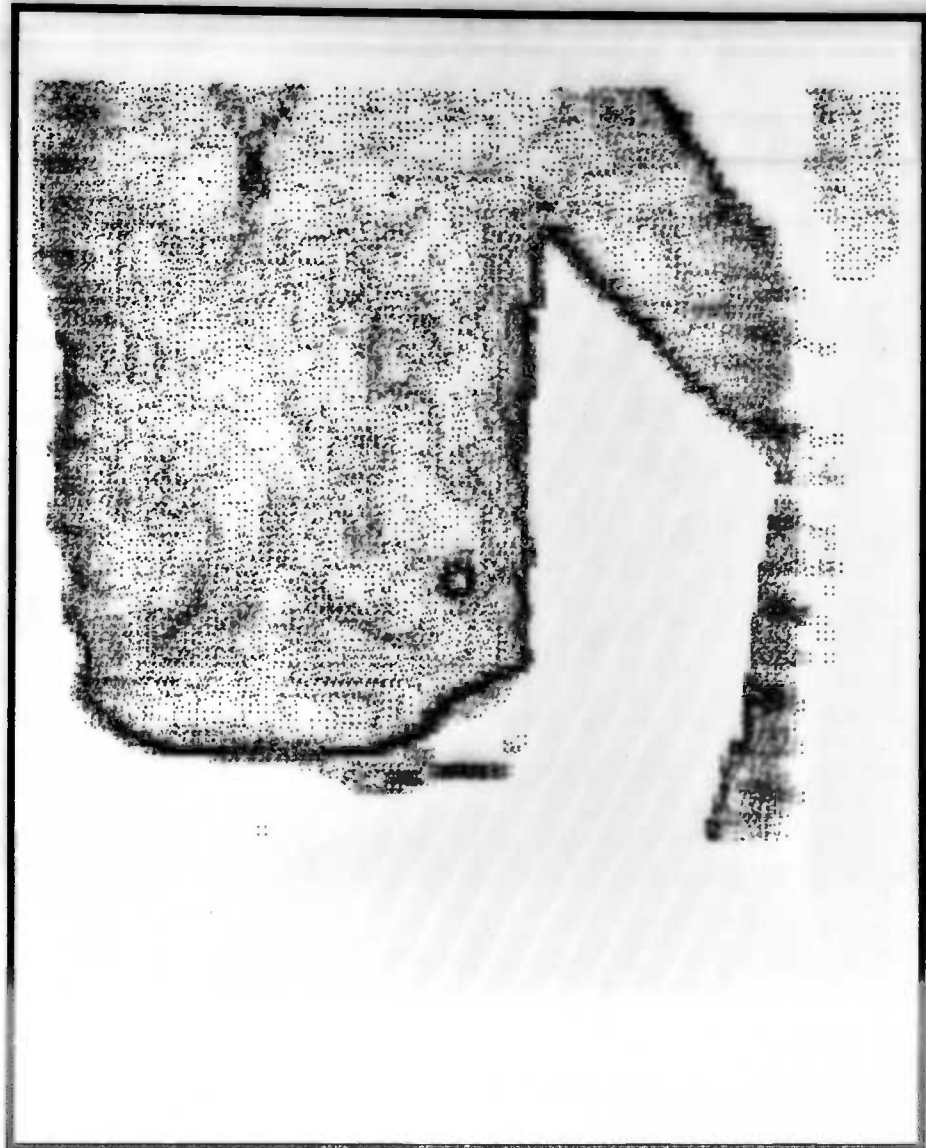


Figure 42. THERM60S ROBERTS.

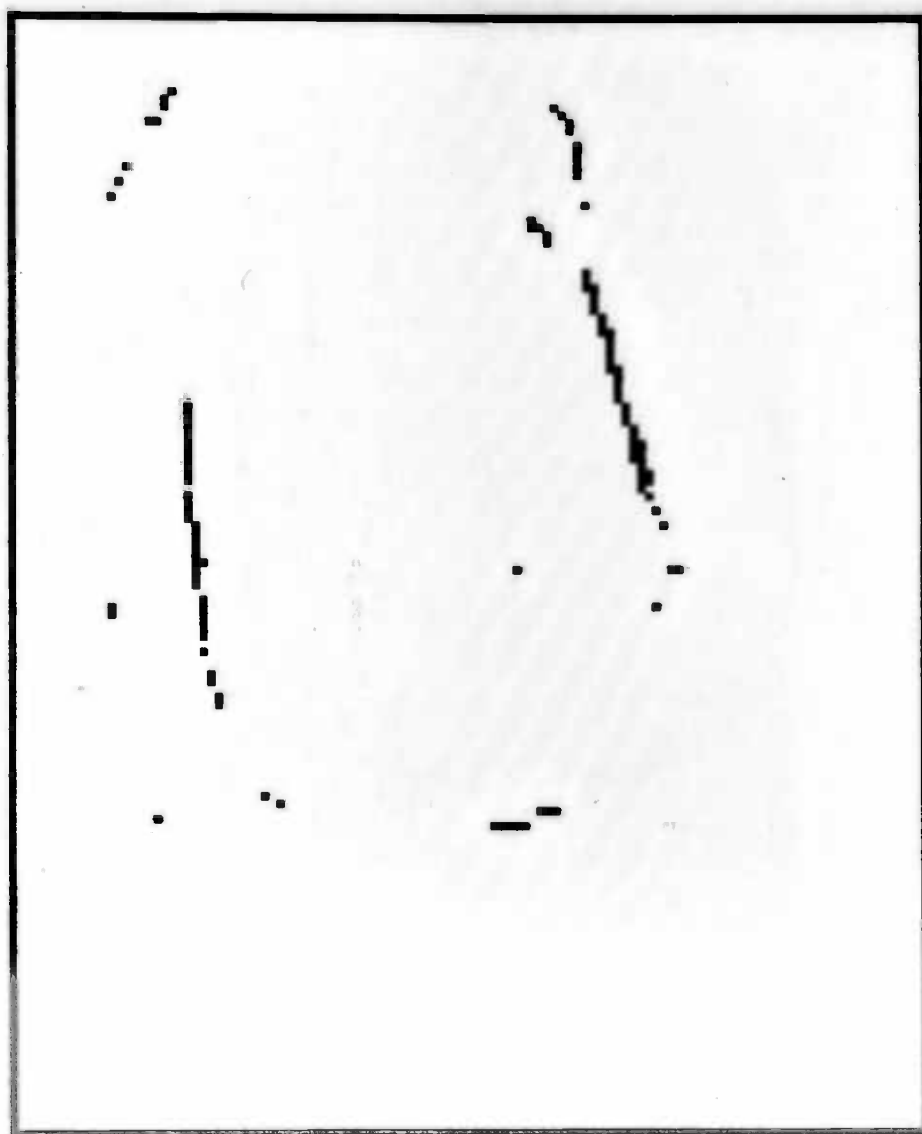


Figure 43. THERM585 ROBERTS + 50% THRESHOLD.

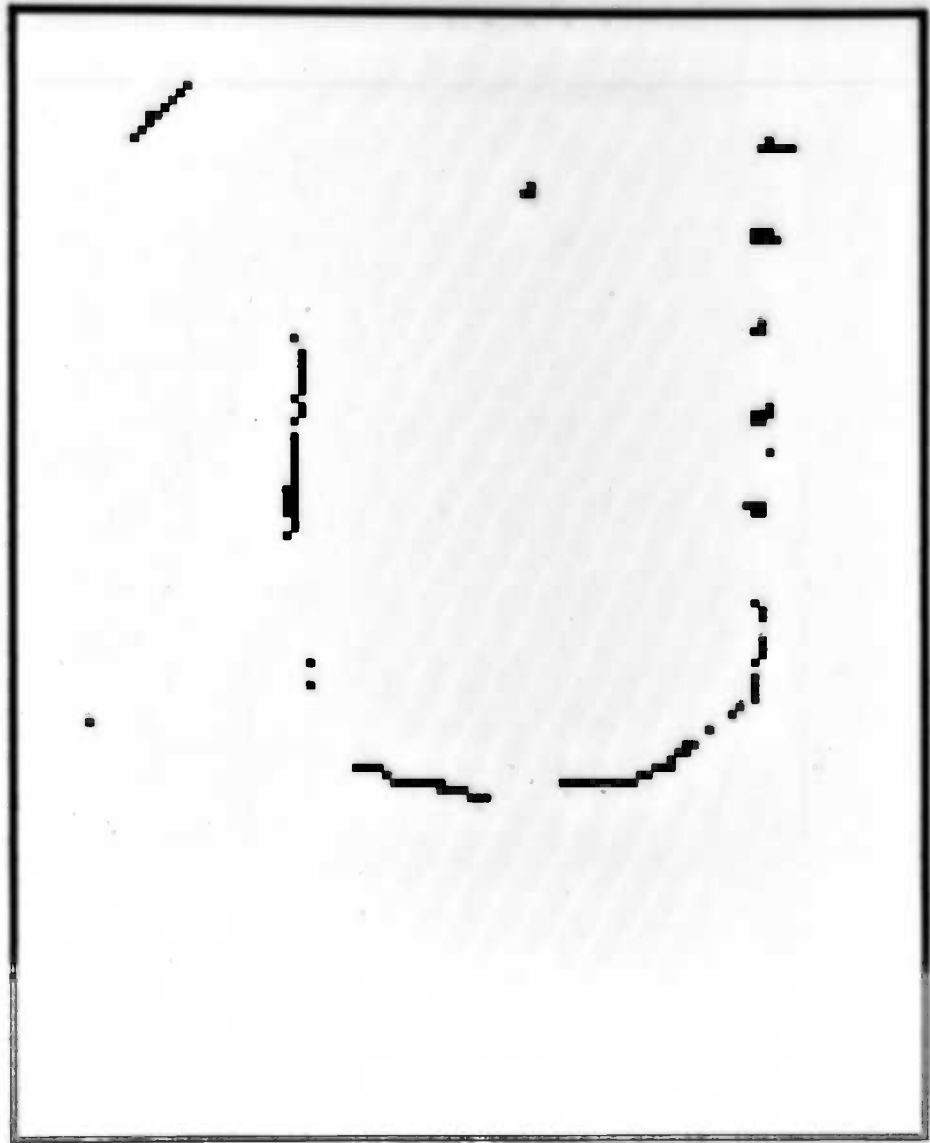


Figure 44. THERM595 ROBERTS + 50% THRESHOLD.



Figure 45. THERM60S ROBERTS + 50% THRESHOLD.



Figure 46. THERM58S. ROBERTS + 25% THRESHOLD.

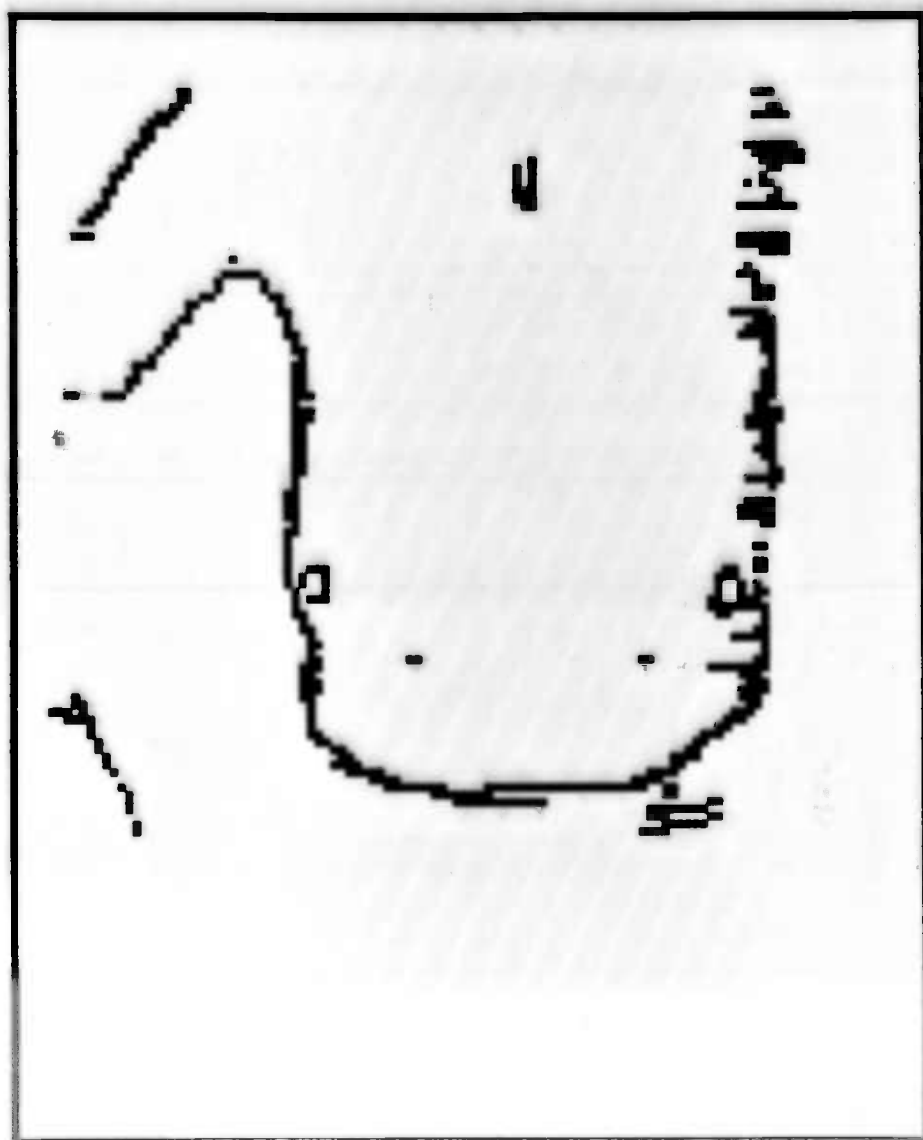


Figure 47. THERMS95 ROBERTS + 25% THRESHOLD.

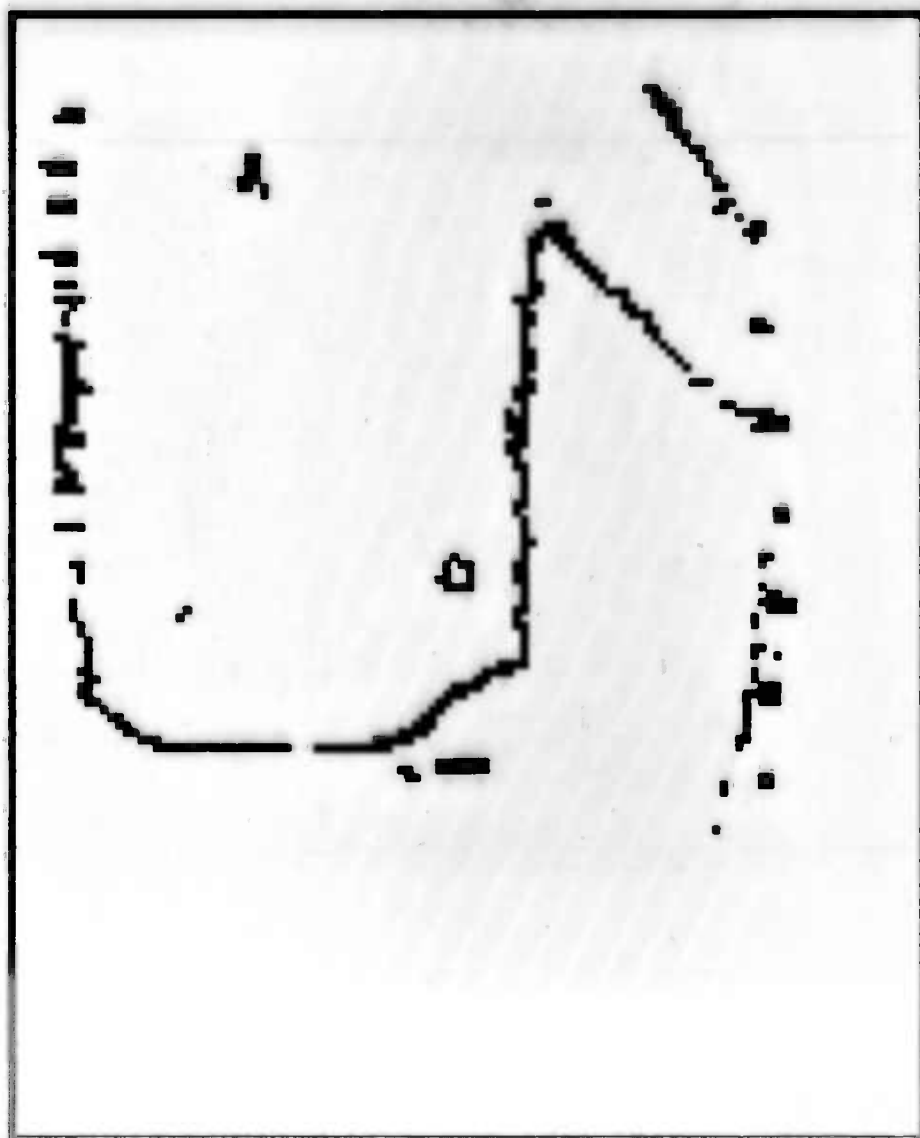


Figure 48. THERM60S ROBERTS + 25% THRESHOLD.

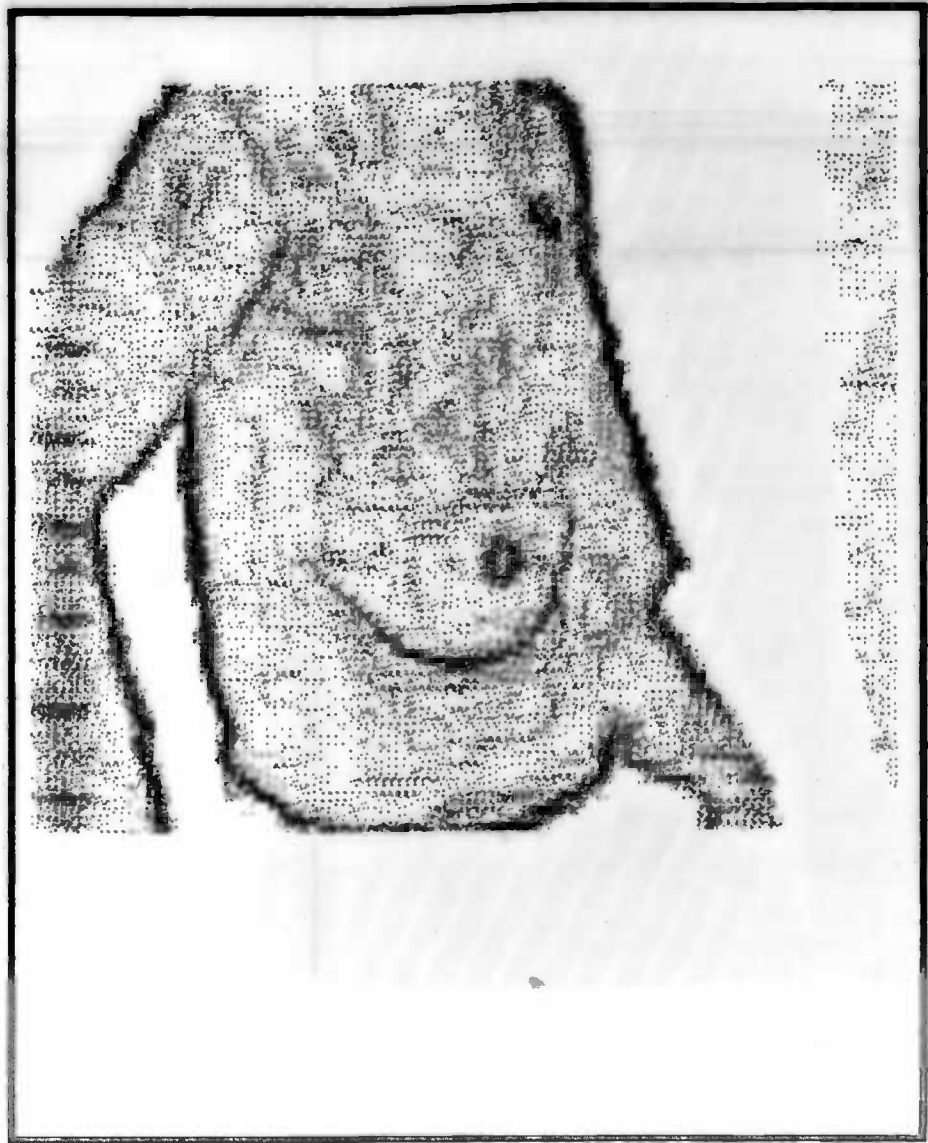


Figure 49. THERMSBS ROBERTM.

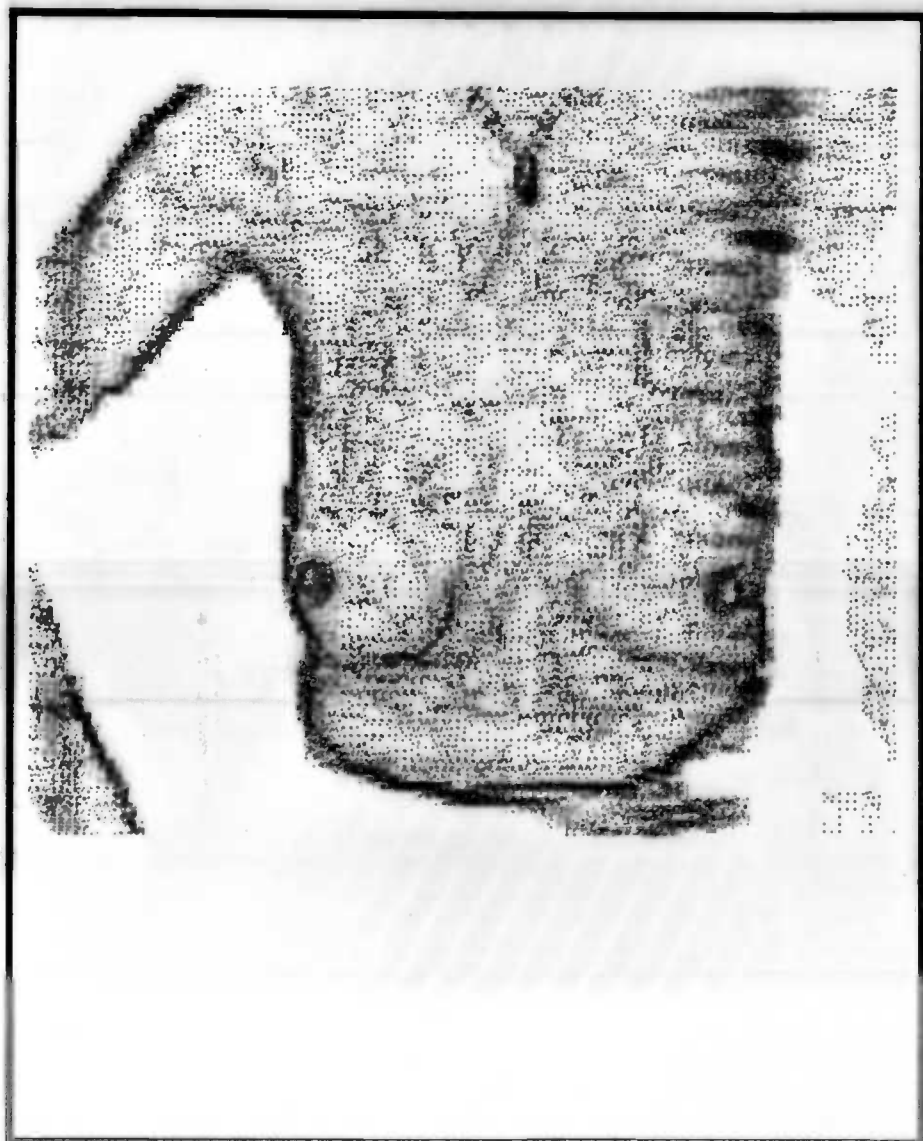


Figure 50. THERMS95 ROBERTM.

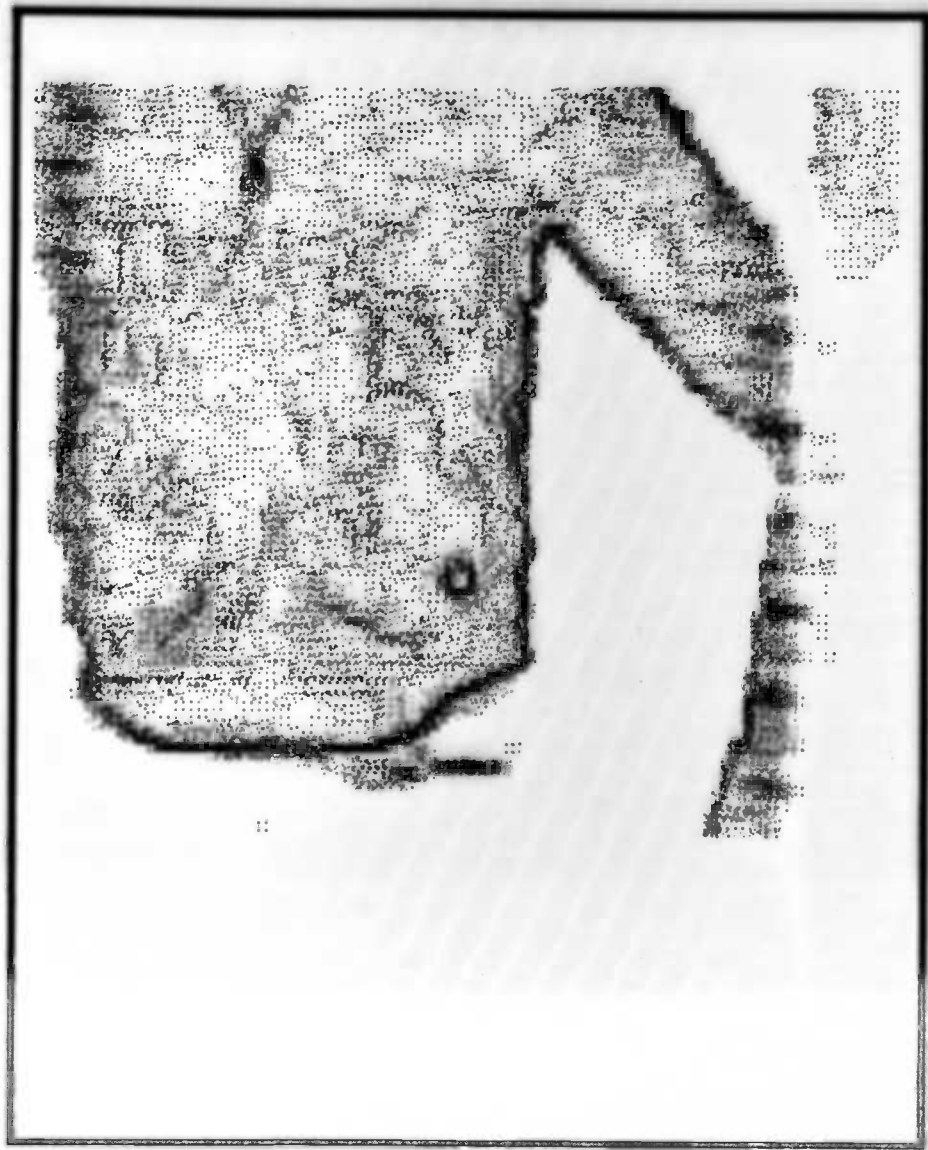


Figure 51. THERM605 ROBERTM.

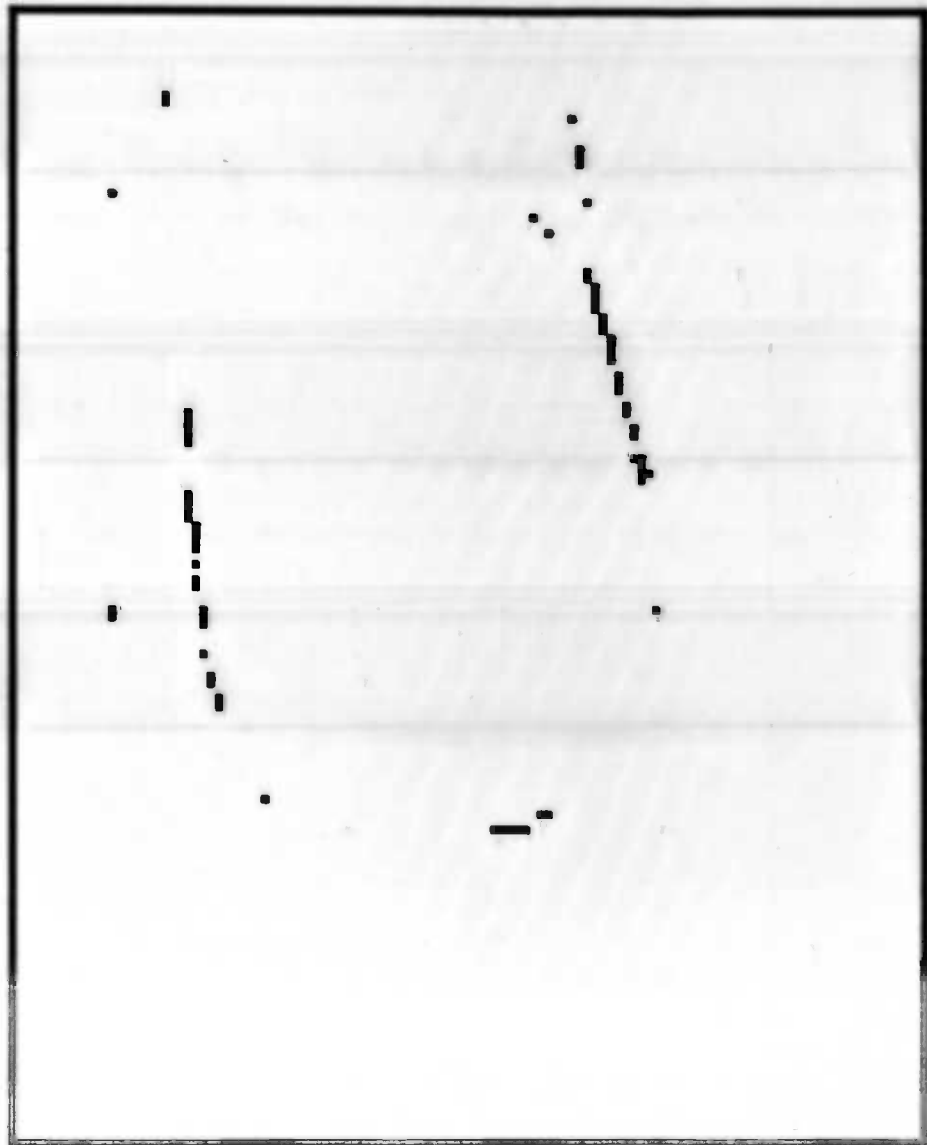


Figure 52. THERM58S ROBERTM + 50% THRESHOLD.

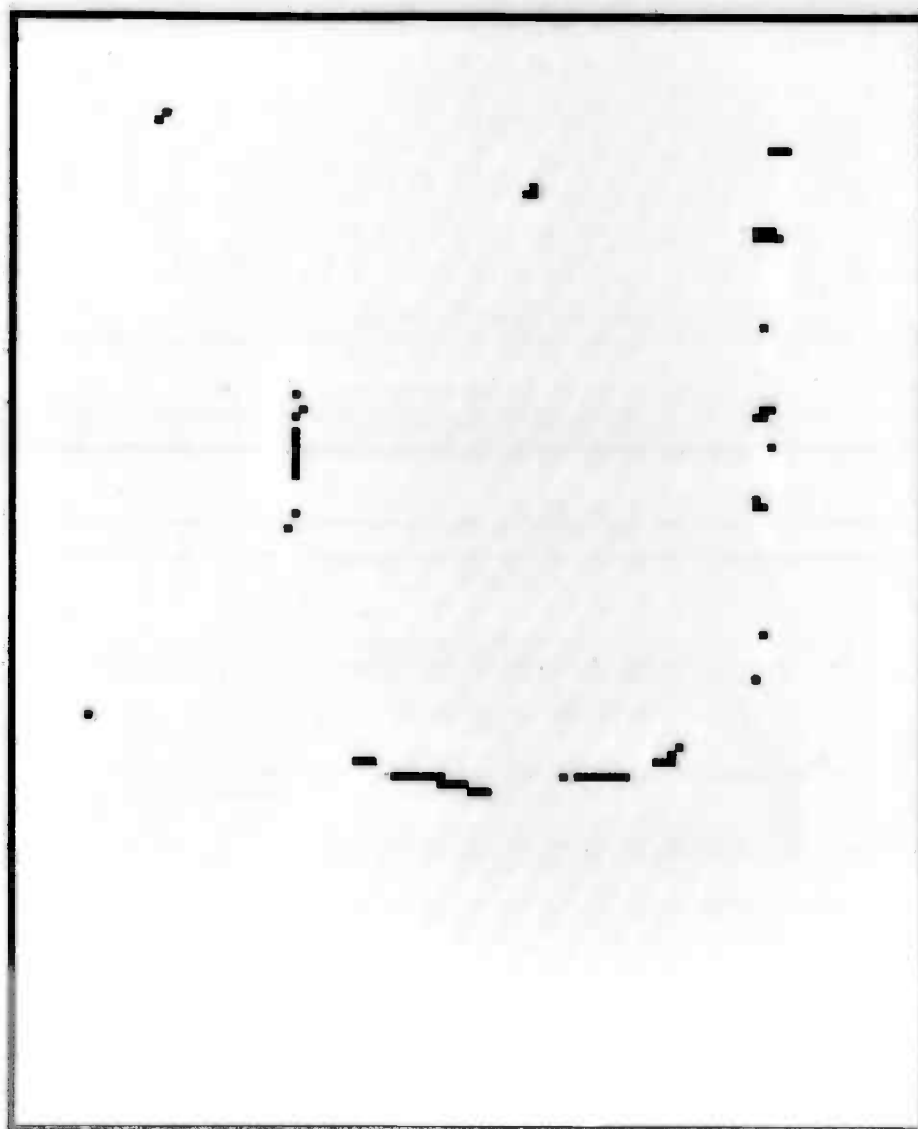


Figure 53. THERMS95 ROBERTM + 50% THRESHOLD.



Figure 54 THERM60S ROBERTM + 50% THRESHOLD.



Figure 55. THERM585 ROBERTM + 25% THRESHOLD.

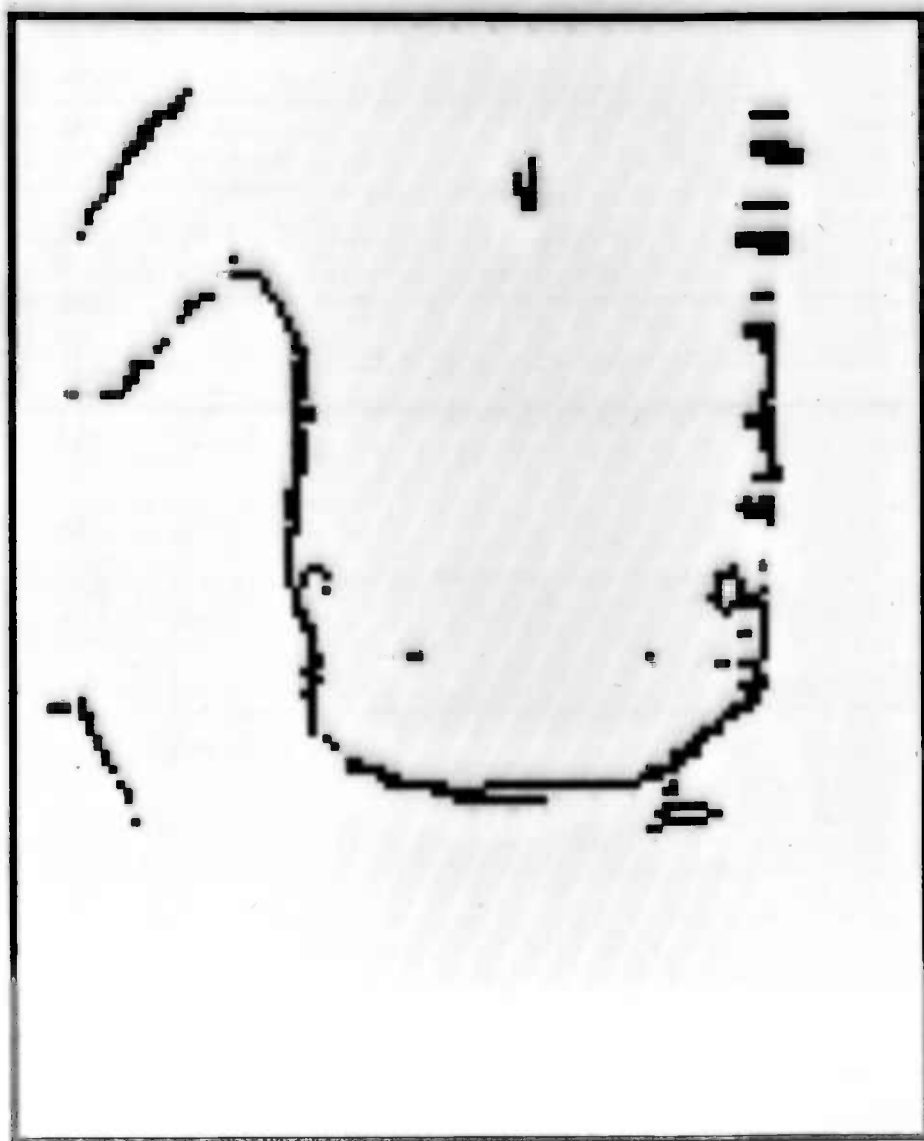


Figure 56. THERM59S ROBERTM + 25% THRESHOLD.



Figure 57. THERM60S ROBERTM + 25% THRESHOLD.



Figure 58. THERM58S SOBEL.

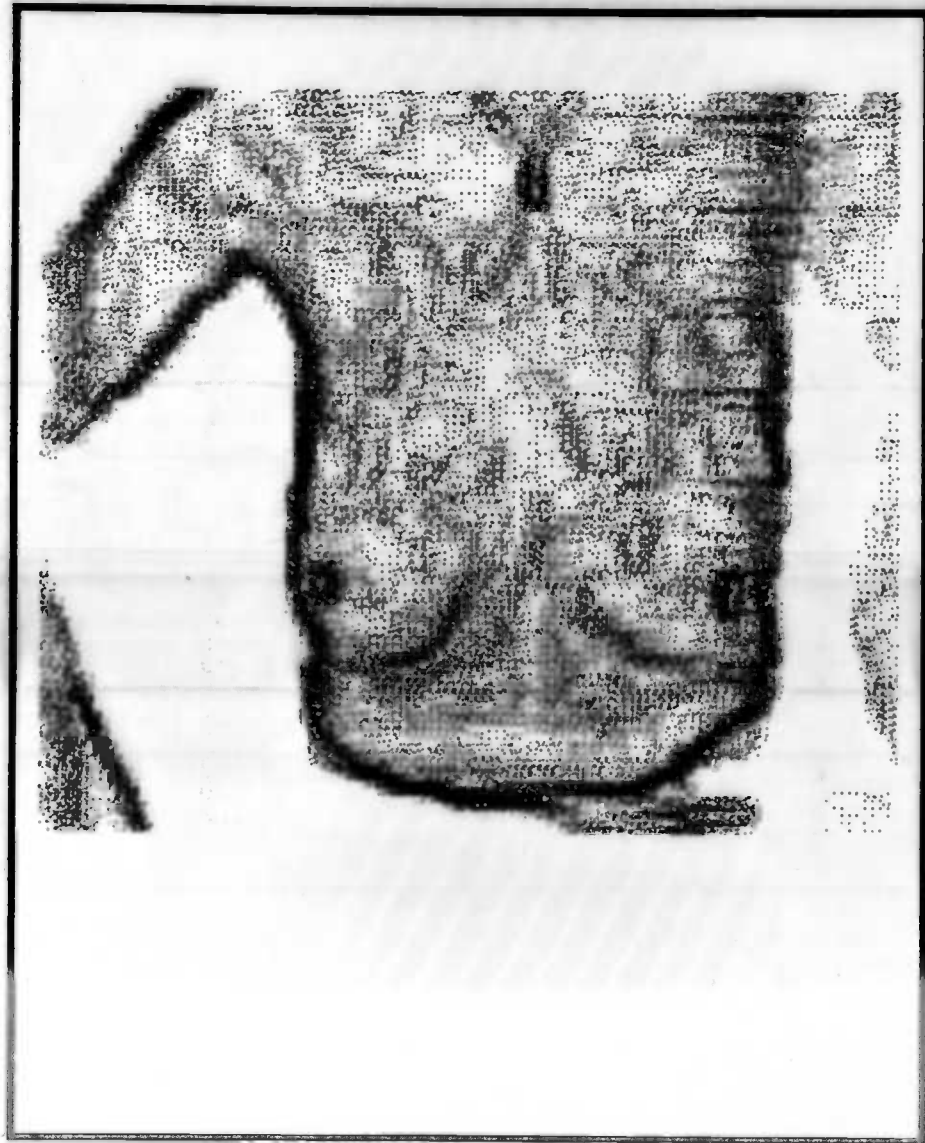


Figure 59. THERM595 SOBEL.

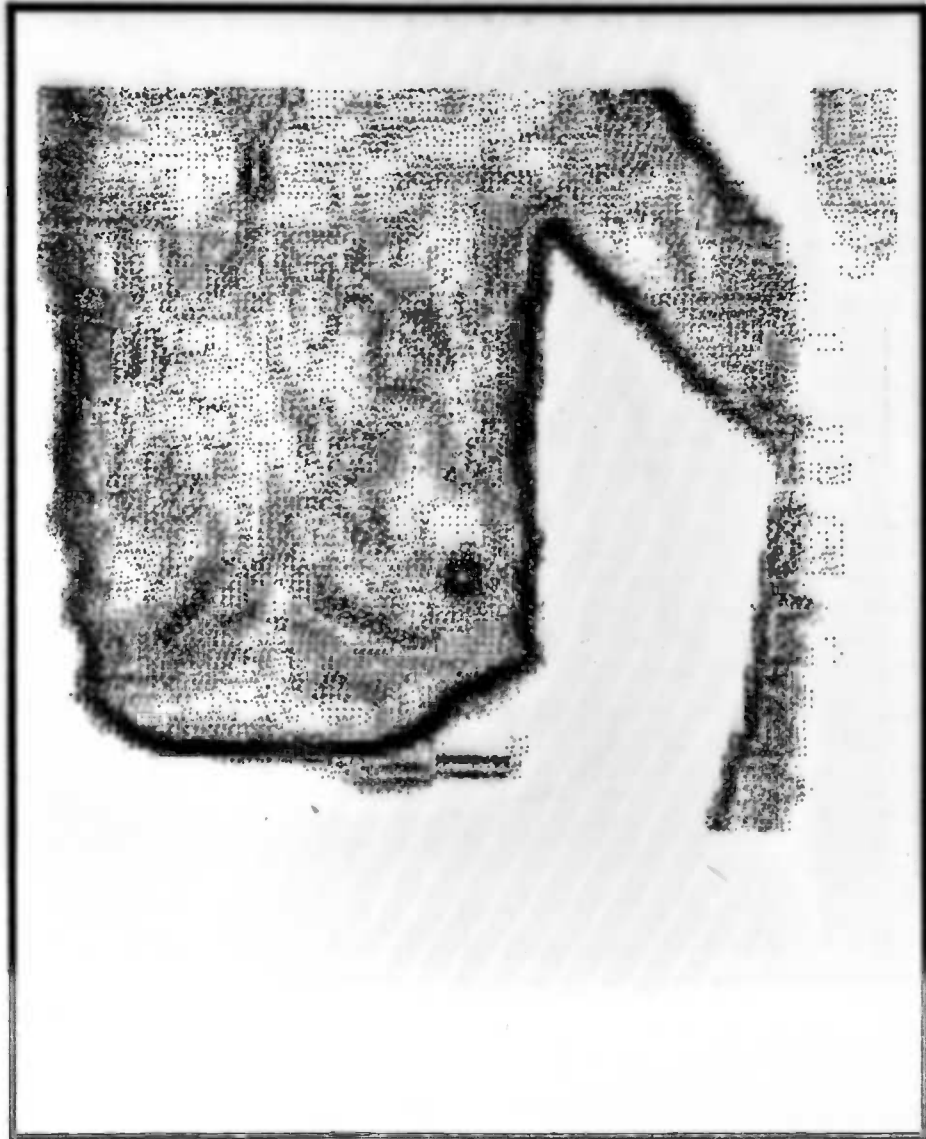


Figure 60. THERM60S SOBEL.

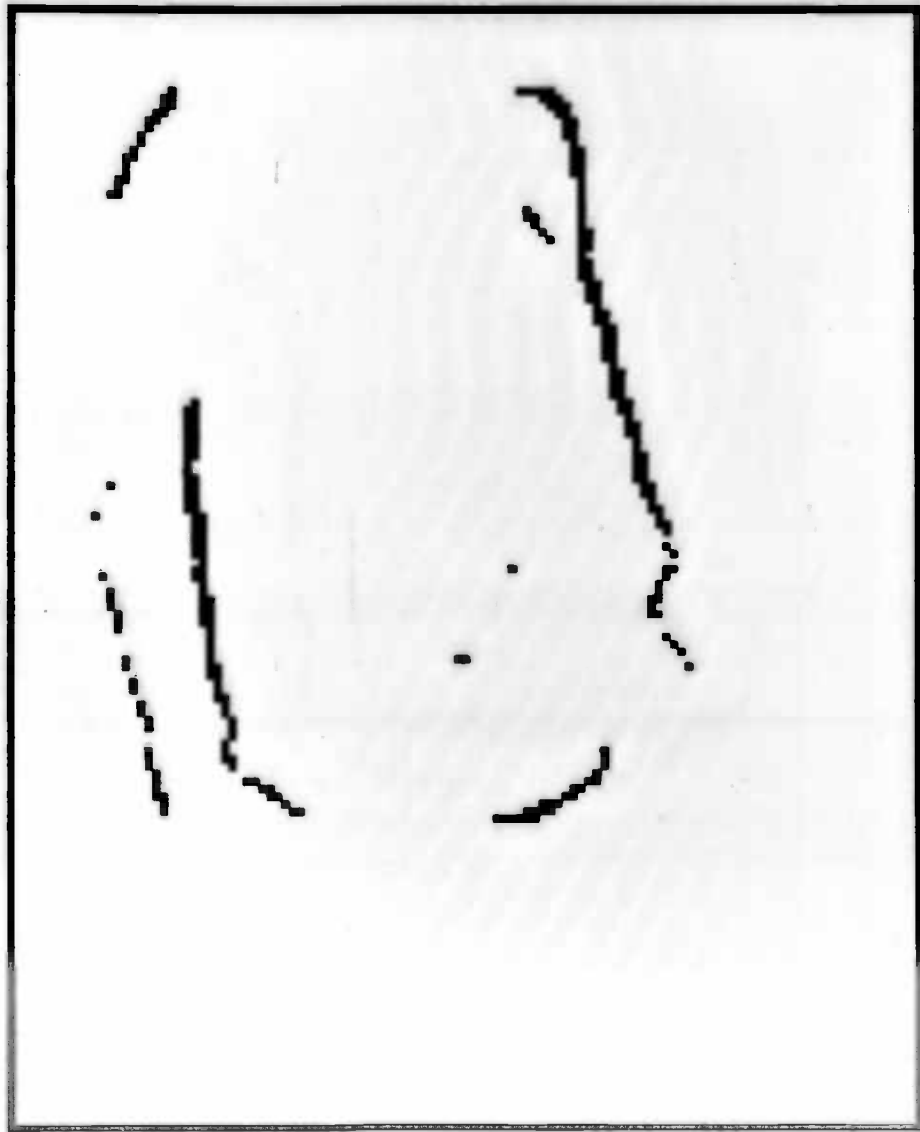


Figure 61. THERM585 SOBEL + 50% THRESHOLD.

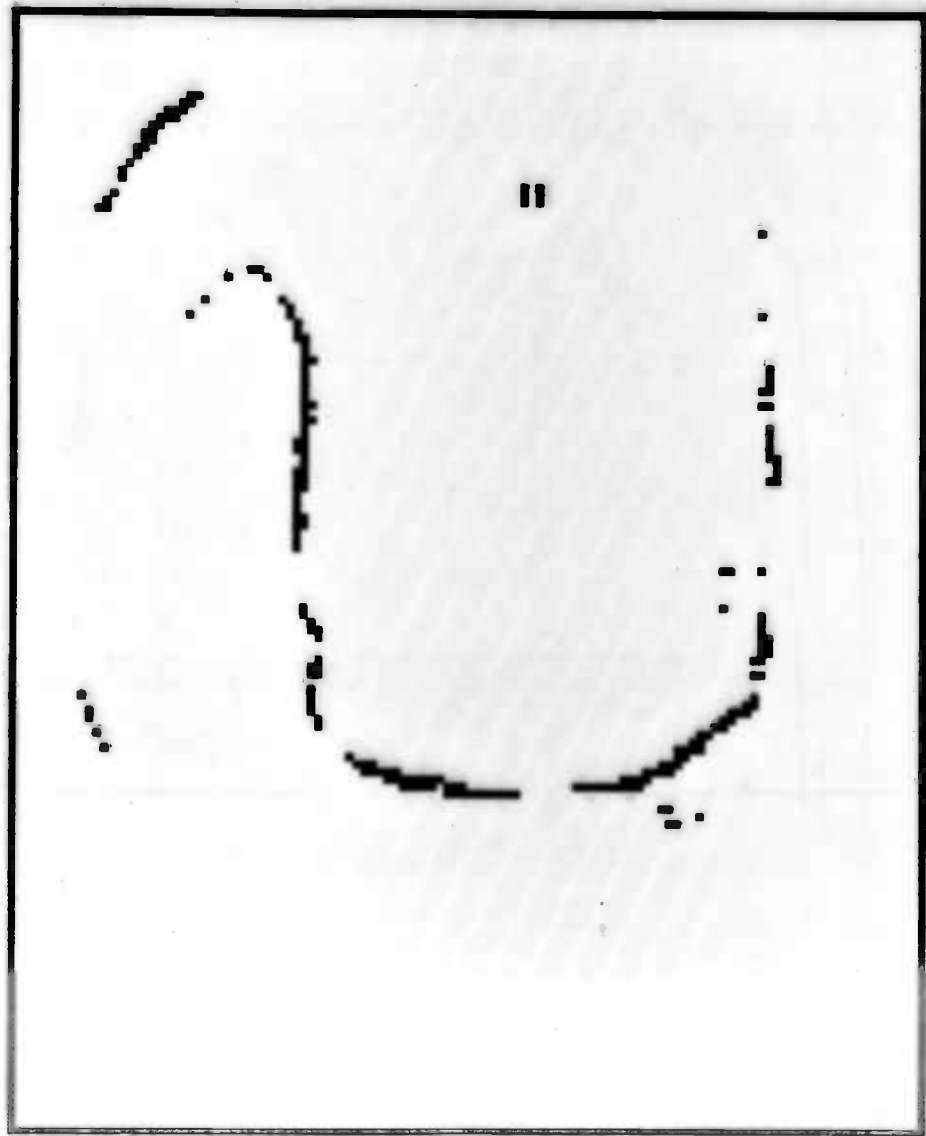


Figure 62. THERM595 SOBEL + 50% THRESHOLD.

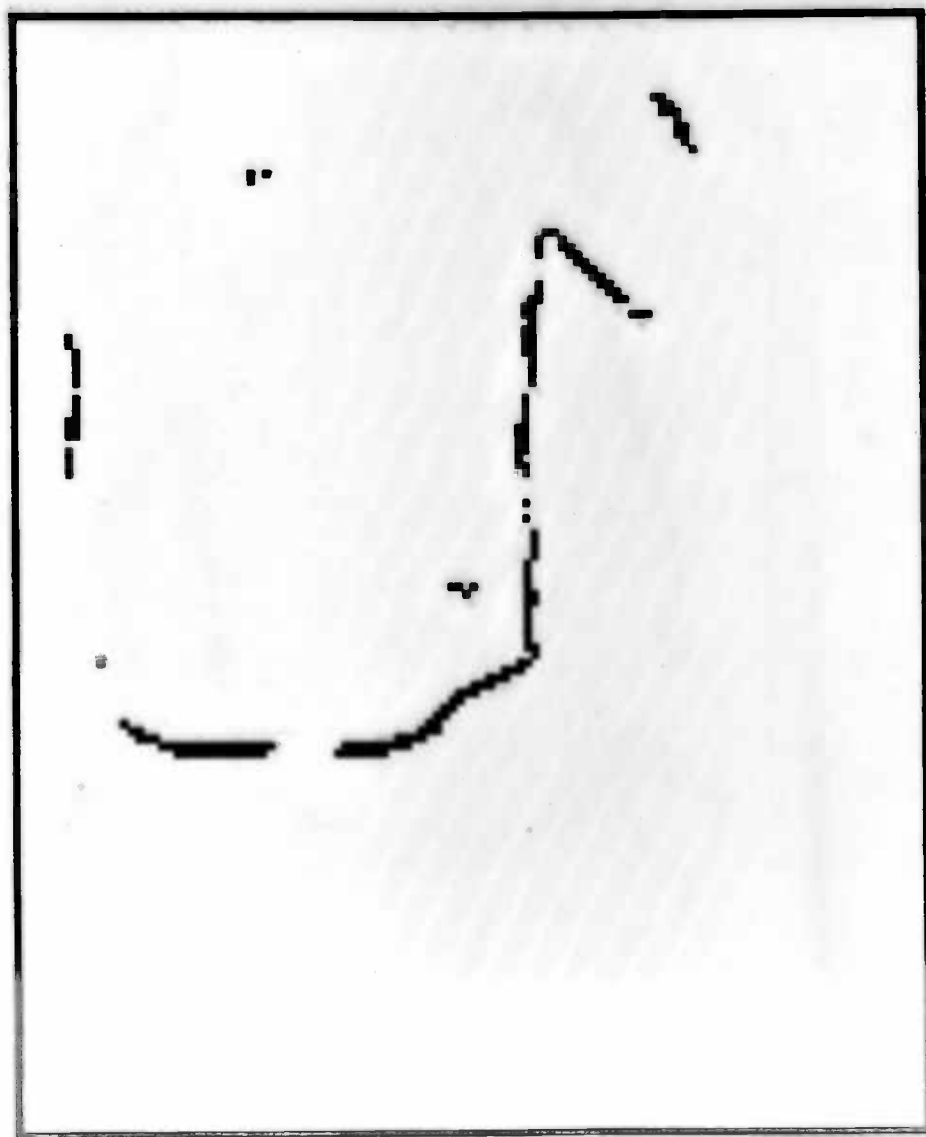


Figure 63. THERM60S SOBEL + 50% THRESHOLD.

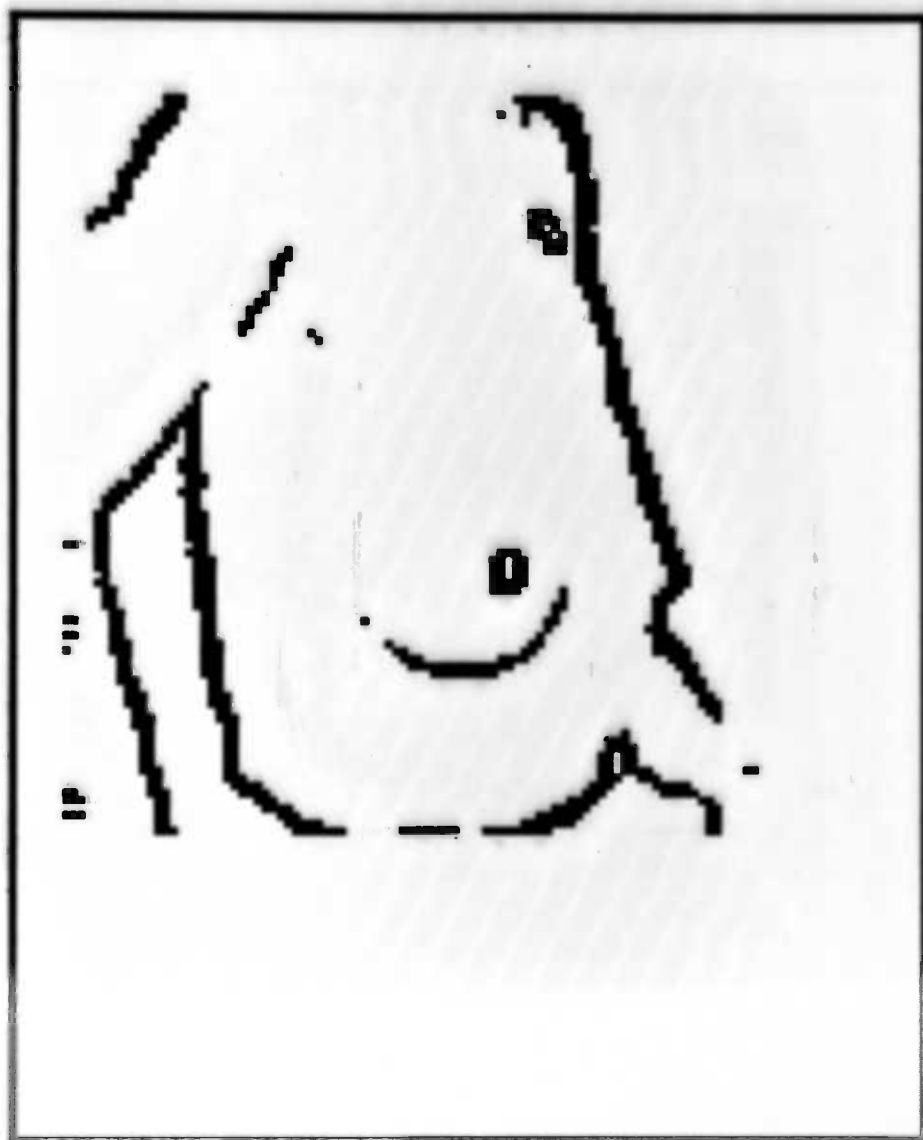


Figure 64. THERM585 SOBEL + 25% THRESHOLD.

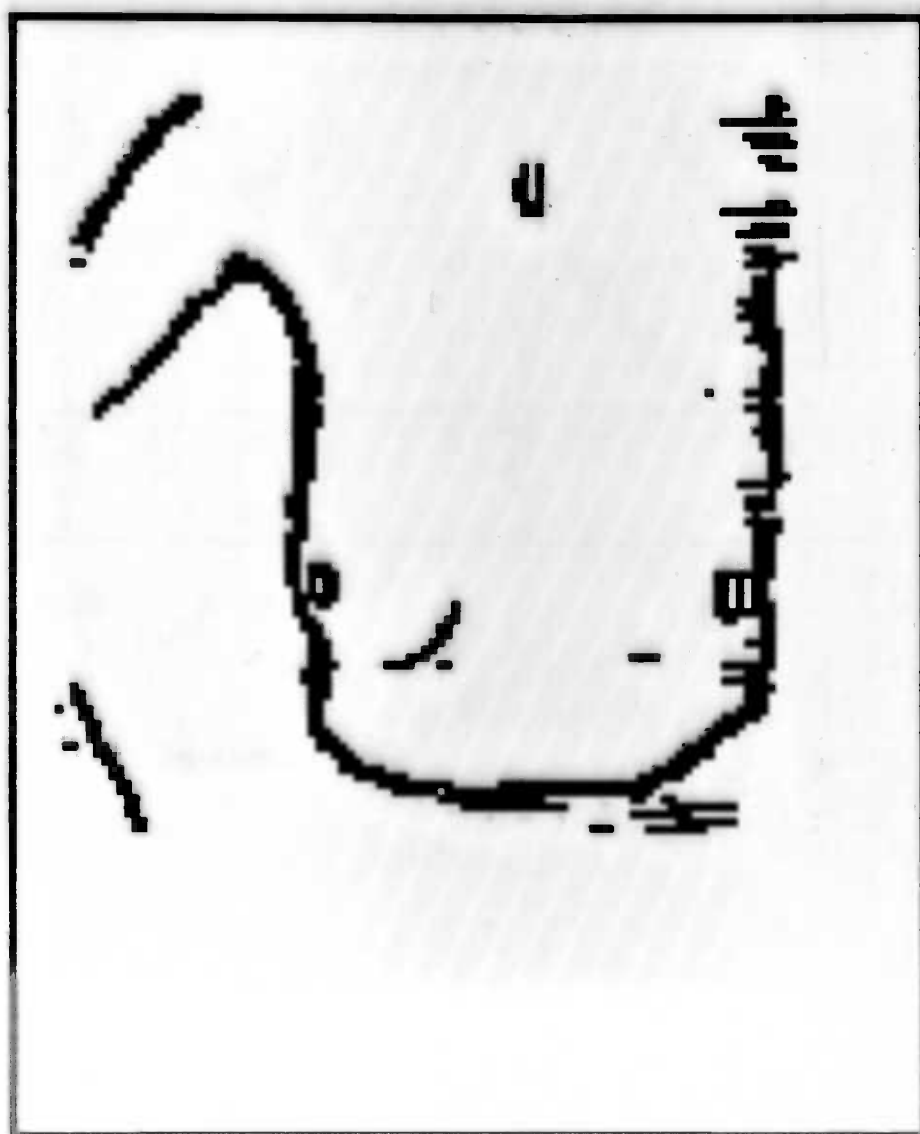


Figure 65. THERM595 SOBEL + 25% THRESHOLD.



Figure 66. THERM60S SOBEL + 25% THRESHOLD.

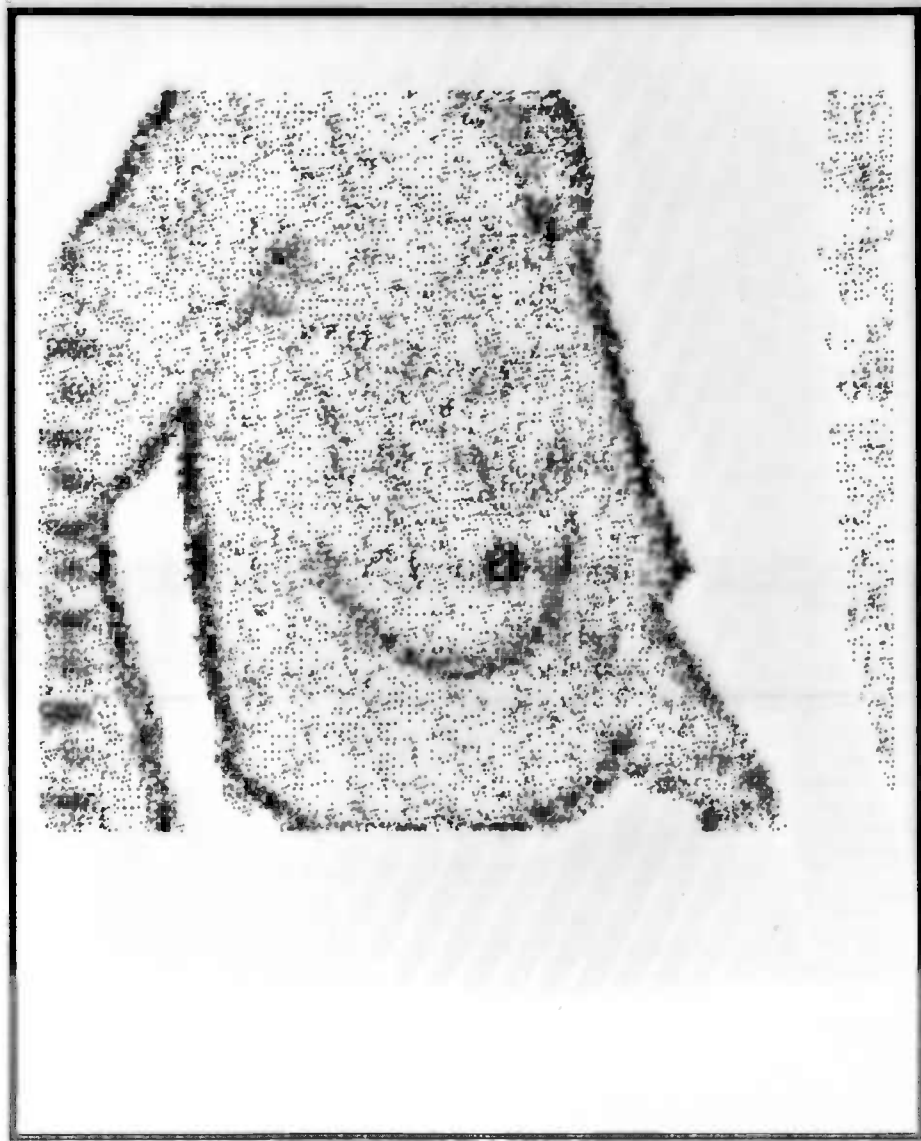


Figure 67. THERM58S LAPLACE.

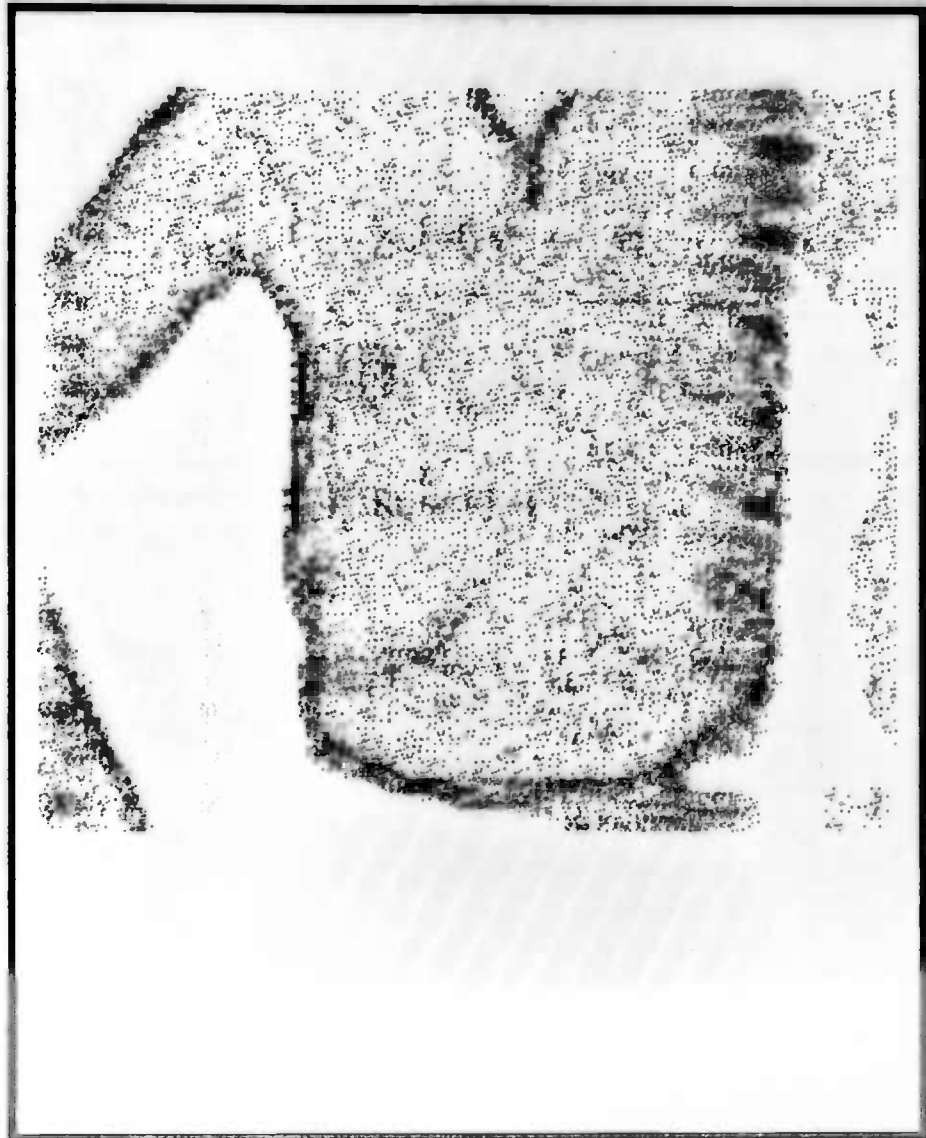


Figure 68. THERM595 LAPLACE.



Figure 69. THERM605 LAPLACE



Figure 70 THERMSBS LAPLACE + 50% THRESHOLD.

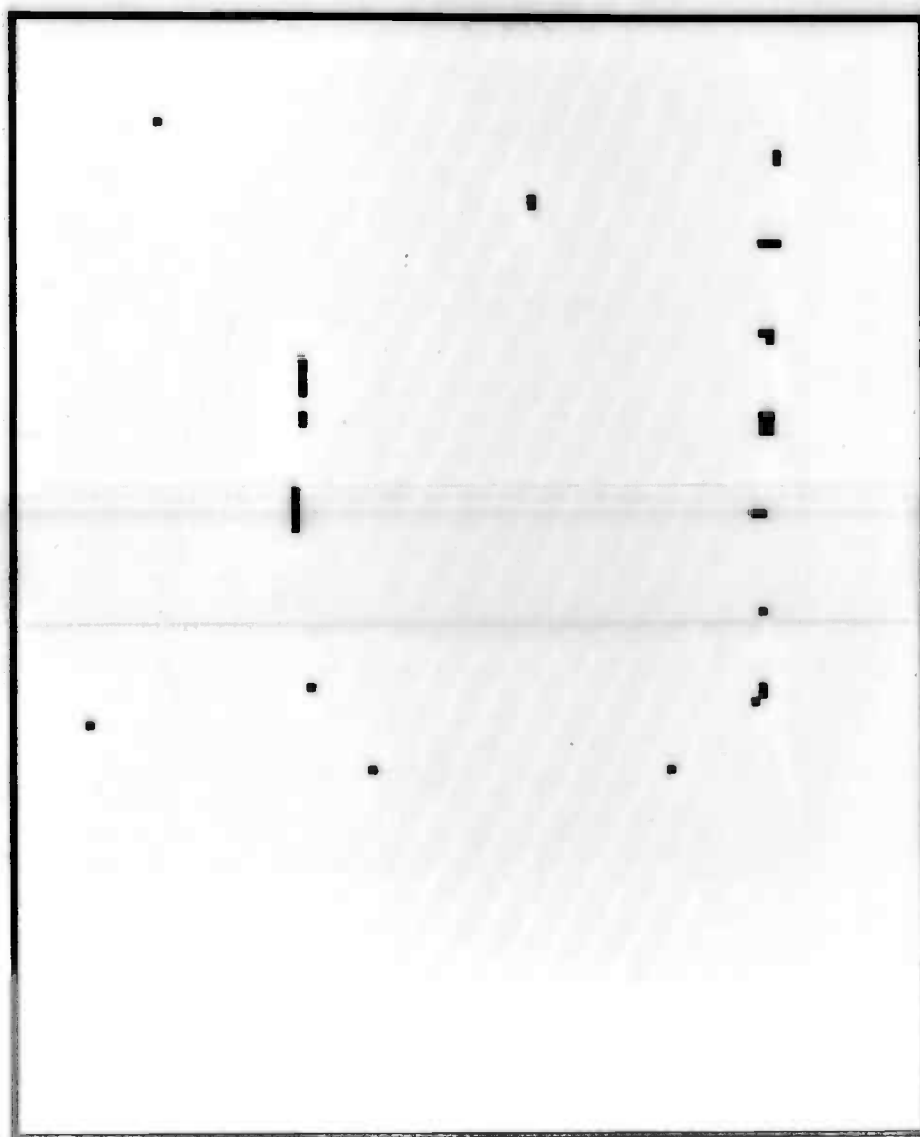


Figure 71. THERM59S LAPLACE + 50% THRESHOLD

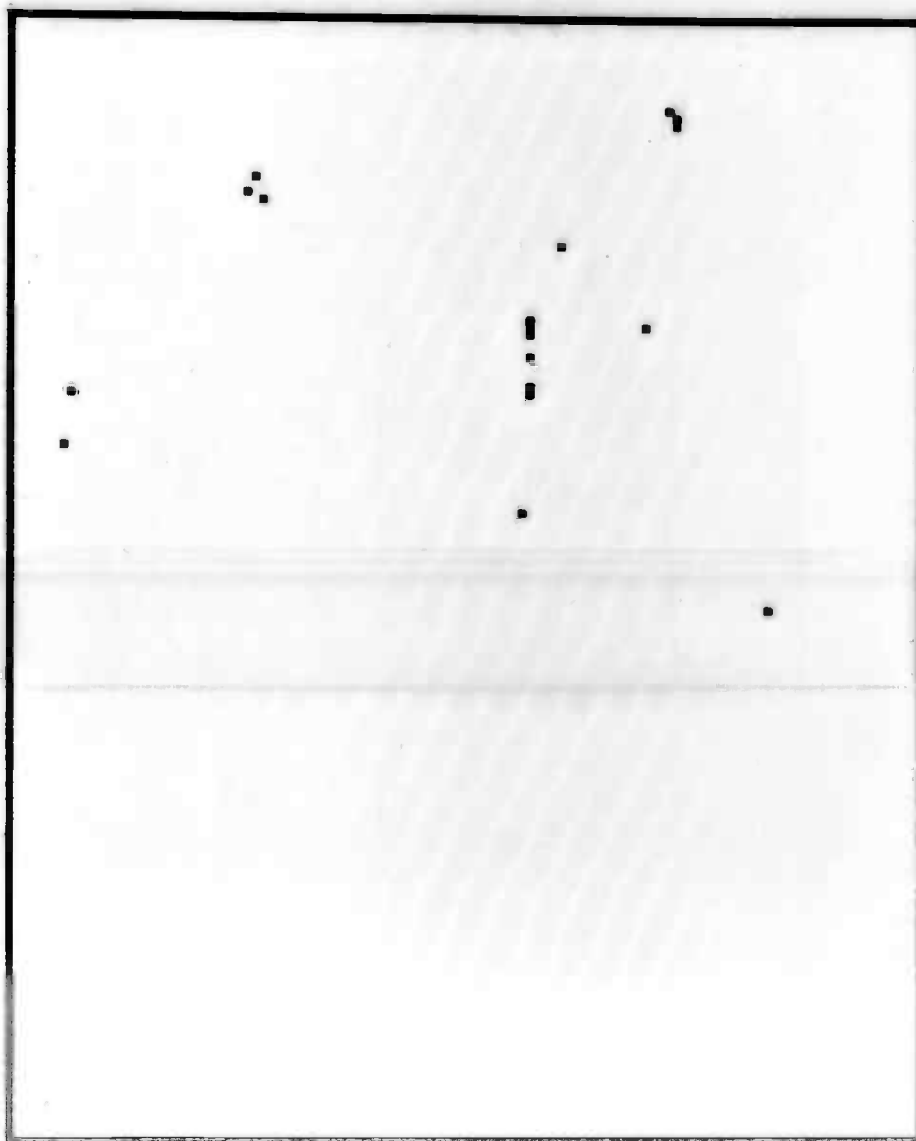


Figure 72. THERM60S LAPLACE+ 50% THRESHOLD.



Figure 73. THERM 585 LAPLACE + 25% THRESHOLD.



Figure 74. THERMS95 LAPLACE + 25% THRESHOLD.



Figure 75. THERM60S LAPLACE + 25% THRESHOLD.

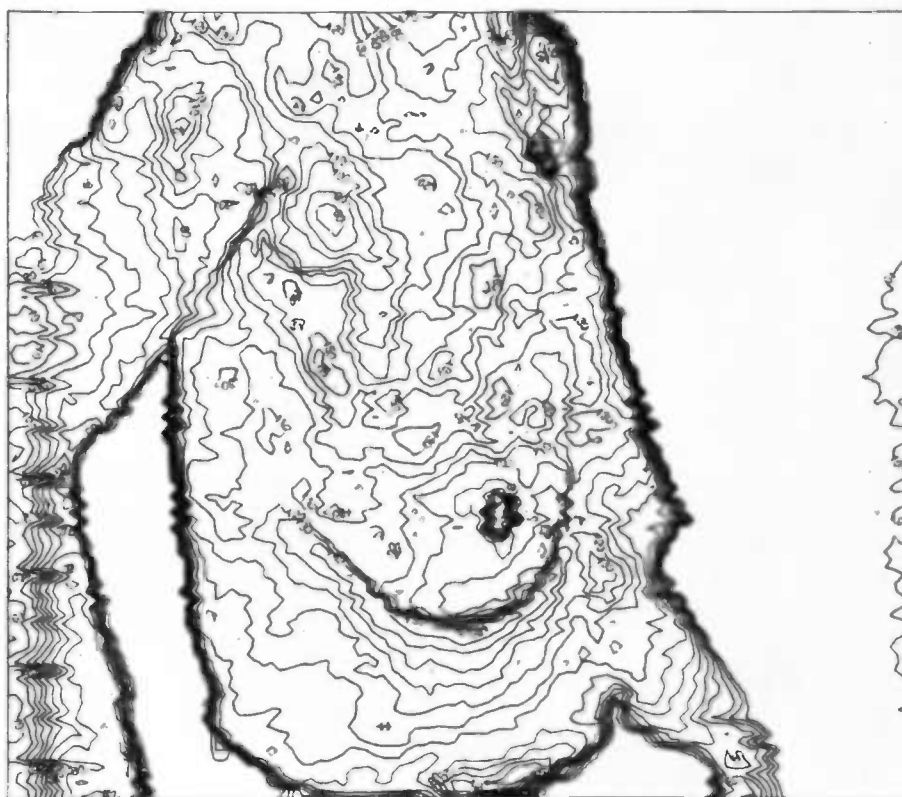


Figure 76. THERM585 CONTOUR.

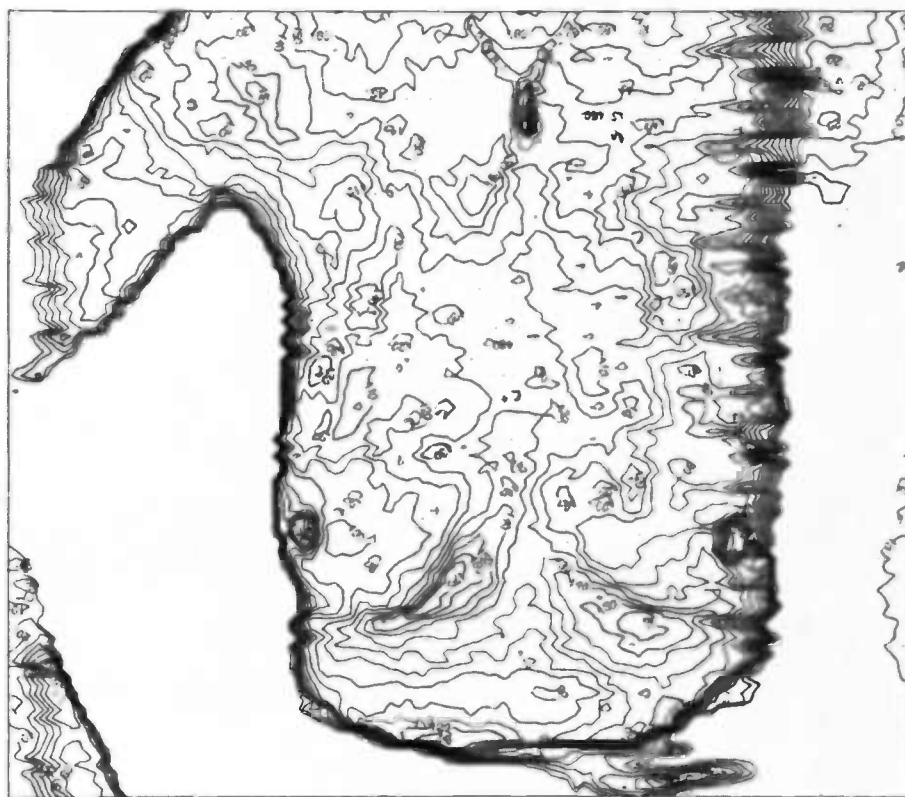


Figure 77. THERMS95 CONTOUR.

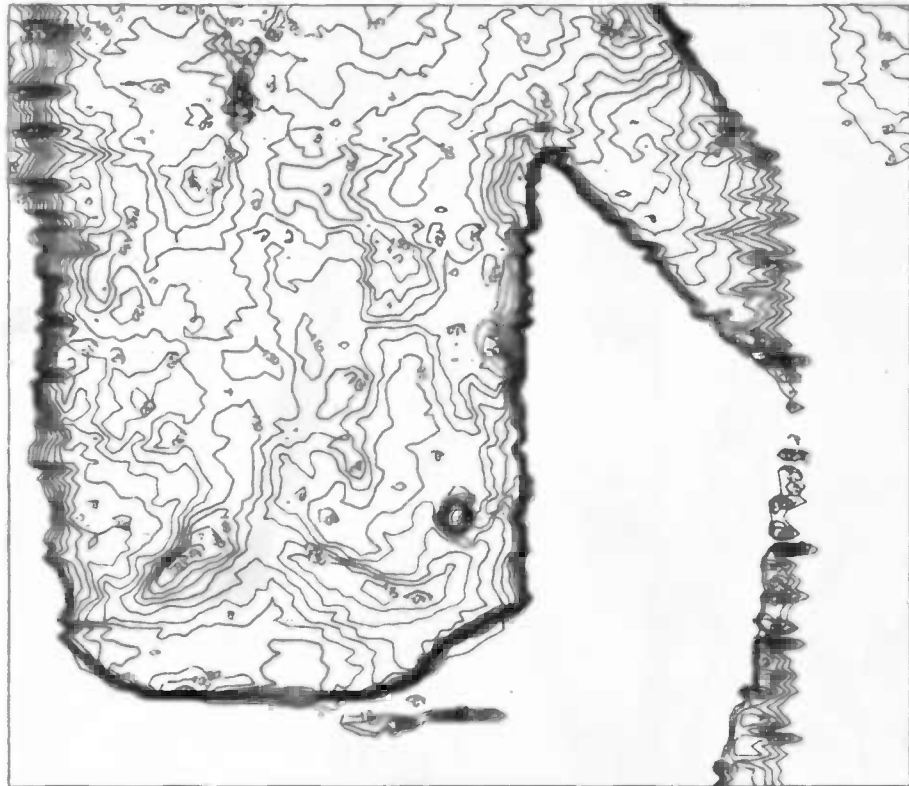


Figure 78. THERM60S CONTOUR.

6. EVALUATION OF TECHNIQUES APPLIED TO THERMOGRAMS.

6.1 Introduction.

6.2 Effectiveness of Algorithms.

6.3 The Usefulness of These Techniques to Breast Screening.

6.1 INTRODUCTION.

This chapter evaluates the effect of different image analysis techniques on breast thermograms. Some conclusions are drawn about which techniques appear to offer the best approach to the automation of breast thermogram interpretation for use in the early detection of breast cancer.

Section 6.2 evaluates the results of the algorithms described in this thesis and section 6.3 discusses their applicability to the automated detection of breast cancer.

Breast thermograms are temperature maps and so any interpretation of these images must be made with care as they must not be confused with true photographic images. No morphological information about the breasts can be derived from these images as we do not see actual breast lesions but rather their effects. Edges in the image are points between homogeneous regions of constant temperature not necessarily between different anatomical parts. From the images analysed by the author it can be seen that temperature

difference enhancement seems to produce quite good body outlines and also picks out the underside of the breast. An operator which both smoothes and differences appears to give the least number of spurious points. Fitting these outline points together to produce a smooth outline of the body and breast would appear to be the next major problem.

6.2 EFFECTIVENESS OF ALGORITHMS.

The principal problem in the automation of quantitative medical thermography is to reduce the amount of subjective region of interest determination and also the amount of subsequent manual computation. This poses the question of how a computer can automatically determine the region and features, i.e. the breast and nipple, for subsequent data processing. From some experience with digital thermographic images it has found that they tend to be noisy but have a reasonably sharp ground to figure contrast, i.e. the background is much colder than the body. There are no straight lines or regular curves in the image this is rather different from a great deal of the work which has previously been done in automatic image analysis. The image does not break up into meaningful homogeneous regions of equal temperature required for region analysis to be successful. There are also at least two different forms of the image to be dealt with, the left and right views. Some of the most important characteristics of the images are:

- 1) There is a marked temperature change at the edge of the body and underside of the breast.
- 2) The nipple is normally a cold region in the breast.

- 3) There is usually only one arm in the image, i.e. on the side of the breast being examined, which could be useful in guiding a search for the breast.

It should also be noted that the angle of the body with respect to the scanner seems to alter the resolution of the breast outline. This can be seen when comparing figures 34, 35 and 36. Further development of image analysis techniques to assist in the delineation of the breast outline would be the development of a line fitting algorithm in conjunction with a breast 'model'. Clearly a computer program needs to know what it is looking for before it can find it. To enable this to be done in a flexible way some high level structural description or model of the object and its relationships to other objects in the image must be developed.

6.3 THE USEFULNESS OF THESE TECHNIQUES TO BREAST SCREENING.

The aim of this work has not been to diagnose breast cancer but to develop techniques for delineating the breast outline to allow subsequent analysis. This further analysis would attempt to classify the breast into a normal or abnormal category which would then be used to assist in the diagnosis of breast cancer. In this thesis, however, the algorithms being considered are only evaluated with respect to their usefulness in delineating the breast outline not with respect to diagnosing breast cancer.

An examination of the images shown in figures 34 to 78 is now made and the relative merits of the algorithms which produced them are discussed. Finally conclusions are drawn about which methods appear most successful for use in attempting to find the outline of

the breast.

1) The Original Images. Figures 34,35,36.

We can see in each of these figures a patient apparently seated in a chair. One common factor in each of these images is that a necklace or pendant that the patient is wearing is just visible. This piece of information was used to conclude that the three images are all of the same patient. (This was necessary because no information was available about each thermogram.) Also common to the three images is banding on both the right and left side of the image. This appears to be as a result of data produced by the scanner, possibly scaling information produced during its fly-back period. The banding could be eliminated by reducing the size of the image by removing the first five or six columns and the last ten or eleven columns.

2) The Scaled Images. Figures 37,38,39.

By scaling and thresholding the images most of the banding has been eliminated. Also in all three images none of the body has been lost although parts of the arm on the periphery of the image tends to disappear. Note that the pendant, nipple and breast outline are all clearer.

3) The results of the ROBERTS operator and thresholding.
Figures 40,41,42,43,44,45,46,47,48.

The application of the ROBERTS operator enhances temperature changes in the images and leaves the body outline very clear. The pendant and nipple are clear but the breast underside is not always clear. The result of thresholding these

images at fifty percent of the maximum value results in some of the body outline points disappearing and the location of the nipple and breast underside likewise disappear. When the threshold level is dropped to twenty five percent of the maximum most of the body outline remains although the breast underside and the nipple do not always remain.

4) The results of the ROBERTM operator and thresholding.

Figures 49,50,51,52,53,54,55,56,57.

The result of applying this operator is qualitatively very similar to applying the ROBERTS operator described in 3 above. However, the results of applying the thresholding at fifty percent and twenty five percent are poorer than the same results for the ROBERTS operator. This is because although the ROBERTS and ROBERTM operators are qualitatively very similar quantitatively they are not.

5) The results of the SOBEL operator and thresholding.

Figures 58,59,60,61,62,63,64,65,66.

The SOBEL operator is a smoothing and differencing operator and as a result it is more successful at finding weak edges without generating spurious edges. The edge of the body can be seen clearly in all three images and in all but one, where it is suggested, the breast underside can be clearly seen. When the fifty percent threshold is applied many of the points can be seen but it is not until the twenty five percent threshold is applied that it is most successful. In all three images at least the start of the breast outline can be seen and the nipple is present in all three.

6) The results of the LAPLACE operator and thresholding.

Figures 67,68,69,70,71,72,73,74,75.

In the three images after the Laplacian has been applied edge points can be seen but they are not very clear. In fact they are often missing completely. The breast underside and the nipple are never very clear. After the fifty percent threshold has been applied only a few spurious points are left. When the twenty five percent threshold is applied the edges can be seen but they have many holes in them.

7) The results of the CONTOUR operator.

Figures 76,77,78.

This operator shows the contour levels in the image in increments of 10 from the coldest points to the hottest points in the image. It clearly shows the outline of the body, the presence of the nipple and the breast underside well. In fact what we are really seeing here is the bunching together of contour levels where the temperature changes rapidly. This is exactly what the edge enhancing operators described above depend on.

Clearly the techniques evaluated above do not form a complete set of all those which can be applied to breast thermograms. By choosing one particular type of technique it has been shown how both the BLIPS package can be used and also how one can select the 'best' or most appropriate technique for a particular kind of data, in this case breast thermograms. The method described above which was obviously the most successful was a combination of the SOBEL operator and the twenty five percent level threshold. It is most consistent at finding weak edges in the image without finding spurious edges. A

more satisfactory method would use an automatic threshold level selector. In fact experiments with such a technique have been carried out but it has not been found to be entirely reliable. The technique works by finding the frequency of occurrence of different values in the differenced image. The idea being that the most common frequencies will be values not associated with edge points. The technique is then to find the lowest threshold level which is above these common points but below the required edge points. Unfortunately this method currently does not regularly and consistently select the correct threshold level. The main failing in this method is that currently it relies on the distribution being bimodal. The level at the minimum point between the two peaks is selected successfully if this is the case. If the distribution is not bimodal then the choice of the correct threshold level is not so simple. For example, if the distribution contains three peaks, which of the two minimum points should be chosen? A fuller specification of the problem is required to produce a suitable algorithm. One possible improvement would be to define the minimum point between the two peaks to be at the maximum possible level.

7. SUMMARY AND PROPOSALS

7.1 Introduction.

7.2 Summary of Thesis.

7.3 Proposals for Future Research and Development.

7.1 INTRODUCTION.

This thesis has been concerned with three areas of research: image analysis techniques, a package for assisting in the design of image analysis systems and a particular problem, breast thermography. In general it has been concerned with the development of a methodology for designing an image analysis system. The classification of image analysis techniques has been to ease the problem of choosing techniques for such a system, and the study of a particular problem has been to test out these ideas in practice.

The background to the field of image analysis has been described and a number of different image analysis techniques have been classified. This classification has been into groups where the common element has been the underlying theory of the techniques or where a similar principle is employed in their operation. An interactive package has also been described which is useful in the design and evaluation of an image analysis system. It allows the researcher to experiment by applying different algorithms and

techniques to his data. The effect of different algorithms on a control group of images has been studied and an evaluation made of the algorithms applied. Finally the design of a particular image analysis system, the computer assisted interpretation of breast thermograms for use in the earlier detection of breast cancer, has been investigated.

The literature supports the contention that early detection of breast cancer results in a decreased mortality rate and that thermography is valuable as a detection method. In view of this and in view of the high incidence of breast cancer, inexpensive mass screening using thermography is extremely desirable. However, limitations of financial and skilled human resources prohibit this with any currently available screening methods. Automated techniques have been presented which view the breast thermogram without human intervention. This paves the way for future research in this area to refine these techniques and experiment with new ones. Classification experiments for establishing precise diagnostic criteria and to conduct a mass screening pilot study with automated interpretation of the breast thermogram could then be carried out.

The remaining two sections in this chapter are a summary of the aims of this research and proposals for future research and development.

7.2 SUMMARY OF THESIS.

As stated earlier, there are no prescriptions for the design of a successful image analysis system. This thesis has attempted to develop a methodology for designing such a system. The approach has been to classify different image analysis techniques into groups to

facilitate the study of the theory behind each technique and at the same time to capitalise on the success of ad hoc techniques. It has been carried out by developing a package which allows the researcher to try out different algorithms on his data and evaluate their effects.

7.2.1 Aims of the Research.

1) In general the aims of this research have been the classification and study of image analysis techniques. When studying the literature on image analysis one is struck by the vast number of papers on the subject and the narrow nature of the topic covered in each paper. The authors of these papers usually advocate very strongly their technique but fail to place it in any general context within image analysis (FU67,KIR71,MAR72,MON71). Nevertheless, many techniques have been successful in their particular application (CHO72,ROB65,CHI74). The important point appears to be that the particular application most strongly affects the choice of algorithms.

Following on from this the development of a package for evaluating and testing image analysis techniques seemed a logical step. Rather than rush blindly into developing yet another 'successful' image analysis algorithm, a package has been designed where the development of such a system would at least be objective to some extent.

2) Specific aims of the research were to study a particular image analysis problem in depth, to apply the package to a particular problem and finally to gain experience in writing image analysis

algorithms. Thermography is typical of a new range of problems in medicine where the data input to the diagnosis is a quantitative picture which has to be compared with another such picture. Thermography is also an attractive tool in that it is non-invasive and could be successfully applied in screening and early diagnosis of breast cancer. The application of the package to the problem of automating the interpretation of breast thermograms has at least initially shown a technique which is successful in enhancing the edges of the body, the underside of the breast and the nipple.

7.2.2 Usefulness of the BLIPS package.

1) In general the development of the package has been useful in that it has allowed the development and testing of ideas, it has provided a number of standard algorithms for easy use and it has simplified the development and testing of new algorithms. It seems crucial not to include the design and testing capability into the actual image analysis system. The effect of doing so is that it is not easy to stop and start again or try different techniques. The system will tend to become static and the 'love at first sight effect' as discussed earlier will retard its development.

2) In particular the package has been useful in allowing the author to experiment with test data and breast thermograms. Some intuitive understanding of the structure of breast thermograms and the affect of noise on data has been obtained by applying many different algorithms. Noise clearly reduces the utility of some of the algorithms but others successfully deal with the noise. An algorithms which clearly enhances edges in the image and is

successful is the presence of noise has been demonstrated. This is a smoothing and differencing operator which, when applied with a thresholding algorithm, picks out edges in the image. An automatic threshold selection algorithm has been experimented with, but has not been entirely successful, especially when the distribution of the values in the image is not bimodal.

7.3 PROPOSALS FOR FUTURE RESEARCH AND DEVELOPMENT.

Below an outline of the direction that future research and development in the areas covered by this thesis should take is given.

7.3.1 Future Research in Image Analysis.

Along with the development of new and improved algorithms the most important area for future research must be the development of a taxonomy for image analysis techniques. Clearly up until now the most successful methods have been ad hoc. The theorists in the field have tended to disregard these successes and concentrate on more 'elegant' or more mathematically sound methods. The vast variety of images to which image analysis techniques can be applied and the vast number of different types of information required from these images suggests that a complete theory is still a long way off.

7.3.2 Future development of the BLIPS package.

The future development of the BLIPS package must be to increase the number and range of different techniques available. This can be done of course by writing more algorithms, but a better approach would be to include other algorithms and procedures acquired from external sources. The Fourier transform is currently the only

'foreign' algorithm but many more could be included from the scientific algorithms packages provided on most computer systems. Currently a particular kind of algorithm missing from the package is that which measures features of the image. For this type of algorithm to be added a more sophisticated method of passing parameters to algorithms would need to be developed if for example feature extraction was to be available.

7.3.3 Future Research and development in breast thermogram interpretation.

The next stage in the analysis of breast thermograms would be to develop a suitable line fitting algorithm. The identification of the outline of the body should be no problem but the identification of the breast will be more difficult. Although only three thermograms have been discussed in this thesis the author has the results of many different algorithms applied to seventy different algorithms (LAI77). The repeatability of the results can be seen in these seventy thermograms. It has been shown that the underside of the breast can be identified but the outlining of the complete breast will require a model of the breast to be defined. For example it is not obvious where the top of the breast is. This will need to be defined by medical staff. Once this model of the breast is defined and the extraction of the breast outline is successful feature extraction from the breast will need to be carried out. This is the measurement of features or characteristics of the breast. Initially the features currently measured manually, e.g. average temperature, maximum and minimum temperature, would be used but more sophisticated measurement techniques could be developed.

7.3.4 Developments in Breast Screening and Thermography.

An evaluation of the success of thermography in breast cancer screening is currently being carried out. Depending of course on the outcome of this research future developments would involve the evaluation of the numerical techniques currently under development. The development of the thermographic scanner should involve the inclusion of a mass storage device for the storage and retrieval of thermograms for further analysis. Two other points which could warrant further consideration are is it possible and of course useful to produce an image of higher resolution and could lower cost equipment be developed using cheaper solid state technology?

Al. BLIPS USER GUIDE.

Al.1 Introduction.

Al.2 Using BLIPS and its File Structure.

Al.3 BLIPS Commands.

Al.4 BLIPS Error Messages.

Al.5 Modifying and Adding Algorithms to BLIPS.

Al.1 INTRODUCTION.

The BLIPS package (Bill Laing Image Processing System) was designed to facilitate the testing and evaluation of image analysis techniques and algorithms. It is principally intended to be used in an interactive mode although it can be used in a batch mode. Data input to the package is of two kinds images, normally held in files, and user commands. The images for input are held in partitioned data files so that identifiable groups of images may be stored together in one file. The output from the package can be directed to an interactive terminal or print file and graphical output can be directed to various graphical output devices, e.g. a graph plotter, a matrix plotter, a visual display unit. The following four sections describe various aspects of using and modifying the package.

The package is currently implemented on the Edinburgh Multi-Access System EMAS and this user guide describes that

implementation.

A1.2 USING BLIPS AND ITS FILE STRUCTURE.

This package has been designed to have a simple ergonomic user interface. This has been done by providing one standard method for input of images to the package and by providing simple 'English' commands to the package. Clear diagnostics and the checking of user supplied parameters is provided.

1) Using BLIPS.

The format of the images for input to the package is described under file structure. The package is called by the user by specifying its name and the name of the partitioned data file containing the images as a parameter.

e.g. BLIPS(PICFILE).

The package and all of the external procedures which it uses are loaded at this point. The package then prompts the user to input a command. The prompt is of the form 'BLIP:' and the user is then free to supply any of the commands which are described in the next section. Error messages as described in section A4 are output to the user in response to a command and the user is then prompted for another or a corrected command.

2) File structure.

Images which are to be input to the package are required to be in a standard format and stored in files in a standard format. As of the order of fifty or sixty images may be required to be accessed at any one time it is convenient to give each image a name and group all of the images of one kind into a file which also is accessed by name. This is done by keeping each

image as a member of a partitioned data file. The partitioned file can be referred to by name and is in fact passed as a parameter to the BLIPS package when it is loaded. This partitioned data file can then hold images each of which can be referred to independently as a member. For example the partitioned data file called PICTURES and an individual image SQUARE within it is accessed as PICTURES_SQUARE. A partitioned data file can contain within it any number of images subject to the maximum file size constraint.

The format of each image within the data file must also be in a standard format. The format is NUMBER OF LINES, LINE LENGTH followed by the values of each point in the image.

i.e.

POINT1 LINE1, POINT2 LINE1, POINTn LINE1

POINT1 LINE2, POINT2 LINE2, POINTn LINE2

.
.

POINT1 LINEm, POINT2 LINEm, POINTn LINEm

Each value is stored in two bytes or a half word this allows image points to have values in the range -32768 to 32767.

A1.3 BLIPS COMMANDS.

This section gives a definition and description of the currently available BLIPS commands. The command language is made up of three types of objects; verbs, data and delimiters. Verbs are command names, e.g. 'GET'. Delimiters are brackets, e.g. '(' and ')', to enclose parameters, commas to separate parameters, e.g. ',' and newline characters or semicolons to separate commands. Parameters are

either integers or strings, e.g. (10,100), (THERM20). Some examples of the use of the command language are:

BLIP:GET(SQUARE)

BLIP:RANGE ; SCALE(50,75)

BLIP:SHADE(30,60)

The currently available commands are:

COMMAND: HARDCOPYON

PARAMETERS: None.

USE: Select the matrix plotter or graph plotter for output.

COMMAND: HARDCOPYOFF

PARAMETERS: None.

USE: Select the interactive terminal for graphical output.

COMMAND: GET

PARAMETERS: 1 string. A member name.

USE: Get the image contained in the member specified from the current partitioned data file into the package's current image.

COMMAND: SAVE

PARAMETERS: 1 string. A file name.

USE: Save the current image to the file specified for subsequent inclusion into a partitioned data file.

COMMAND: FINISH

PARAMETERS: None.

USE: Stop the package and return to the system command level.

COMMAND: PRINT

PARAMETERS: 1 string. A filename or an output device name.

USE: Print the current image to a file, i.e. as numerical values
formatted line by line.

COMMAND: SIZE

PARAMETERS: None.

USE: Print the number of lines and the length of each line in the
current image.

COMMAND: RANGE

PARAMETERS: None.

USE: Print the maximum and minimum values in the current image and
also their positions within the image, i.e. which line and
column they are in.

COMMAND: SHADE

PARAMETERS: 2 integers. The first specifying the value of white and the second specifying the value of black in the current image.

USE: Output a shaded version of the image to the current output device. With the parameters defining any scaling which is to be done.

COMMAND: THRESHOLD

PARAMETERS: 2 integers. Defining a range in the current image.

USE: Map all of the points which are within the defined range into 1 and all other points into 0.

COMMAND: SCALE

PARAMETERS: 2 integers. The first defining a new zero value in the image, i.e. this value is to be subtracted from each image point, and the second defining a scaling factor as a percentage, i.e. multiply each image point by $n/100$.

USE: Shifts and scales the image.

COMMAND: NEGATE

PARAMETERS: 2 integers. The first specifies a minimum value and the second a maximum value.

USE: Produces a negative of the current image in the range specified.

Note that values greater than the second parameter are mapped into the value of the first parameter and vice versa.

COMMAND: ROBERTS

PARAMETERS: None.

USE: Applies the Roberts cross operator to the current image.

COMMAND: ROBERTM

PARAMETERS: None.

USE: Applies a more efficient implementation of the Roberts cross operator.

COMMAND: SOBEL

PARAMETERS: None.

USE: Applies a smoothing and differencing operator.

COMMAND: LAPLACE

PARAMETERS: None.

USE: Applies a Laplacian operator to the current image.

COMMAND: SHRINK

PARAMETERS: 1 integer. A shrinking factor for the image.

USE: Reduce the size of the current image by the amount specified.

COMMAND: WINDOW

PARAMETERS: 4 integers. The coordinates of a section of the current image.

USE: Extracts from the current image a section specified by the coordinates of the bottom left corner and the upper right corner.

COMMAND: SANDPNOISE

PARAMETERS: 4 integers. The first specifying the level of white in the current image, the second specifying the level of black, the third specifying the level above which noise is to be added and below which noise is to be subtracted and finally the percentage of points which are to be modified.

USE: Adds salt and pepper noise to the image.

COMMAND: FOURIER

PARAMETERS: None.

USE: Apply a Fourier transform to the image. Leaving the result in two arrays the real and imaginary parts.

COMMAND: IFOURIER

PARAMETERS: None.

USE: Apply an inverse Fourier transform to the two arrays. Leaving the result in the current image.

COMMAND: SHADEFOURIER

PARAMETERS: None.

USE: Output to the current output device the magnitude of the two arrays resulting from the Fourier transform. The scaling used is logarithmic.

COMMAND: FTEMPLATE

PARAMETERS: 2 strings. Two member names.

USE: Apply a Fourier transform to both images point multiply the two transforms, apply an inverse transform and leave the result in the current image.

COMMAND: CIRCLES

PARAMETERS: 3 integers. The first a radius of a circle, the second a threshold level and the third an angle of search.

USE: Find the centres of circles with the specified radius, using the threshold specified to ascertain edge points and only search within the angle specified. The number of times a circle centre is found is accumulated in the current image at the centre point.

COMMAND: CONTOUR

PARAMETERS: 3 integers. A start level, an increment and a stop level.

USE: Output to the current output device the contour levels as specified.

A1.4 BLIPS ERROR MESSAGES.

There are two types of error message output by the BLIPS package. The first type is output by the control routine of the package and normally refers to the most recent command typed by the user. It may also refer to the validity of the parameters specified for the command. The second type of error is output by the procedures called by the control routine and usually refer to the range or type of the parameters passed to it. The second type always identify which procedure they are produced by.

TYPE 1 ERRORS:

WHAT?

An invalid command has been entered.

PARAMETERS?

Insufficient parameters or parameters of the wrong type have been supplied with the command just entered.

IMAGE TOO LARGE

The image which is being 'GOT' from the partitioned data file is too large for the current image.

NO IMAGE IN MEMORY

There is no current image to apply the command just entered to. i.e. No image has been 'GOT' yet.

IMAGE NOT IN FILE

The image which is being 'GOT' is not contained in the partitioned data file.

NOT A VALID MEMBER NAME

The name of the image supplied is not of the correct format.

NOT A VALID FILE NAME

The name of the file supplied is not of the correct format.

TYPE 2 ERRORS.

CONTOUR: PARAMETERS INSIDE OUT

The parameters passed to the contour routine do not satisfy the conditions $PARAM1 \leq PARAM3$ AND $PARAM3 = n * PARAM2 + PARAM1$.

NEGATE: PARAMETERS INSIDE OUT

The parameters to the negate routine do not satisfy the condition $PARAM1 < PARAM2$.

SHRINK: FACTOR TOO LARGE

The result of applying the shrink routine would be to leave only 1 column or 1 line in the current image.

WINDOW: BAD PARAMETERS

The parameters passed to the window routine do not satisfy the condition $1 \leq PARAM1 \leq \text{NUMBER OF LINES}$ AND

$1 \leq PARAM2 \leq \text{LINE LENGTH}$ AND

$1 \leq PARAM3 \leq \text{NUMBER OF LINES}$ AND

$1 \leq PARAM4 \leq \text{LINE LENGTH}$ AND

$PARAM1 < PARAM3$ AND $PARAM2 < PARAM4$.

THRESHOLD: PARAMETERS INSIDE OUT

The parameters passed to the threshold routine do not satisfy the condition $PARAM1 \leq PARAM2$.

SANDPNOISE: BAD PARAMETERS

The parameters passed to the salt and pepper noise routine do not satisfy the condition $PARAM1 < PARAM2$ AND

$PARAM1 \leq PARAM2 \leq PARAM3$

AND $0 < PARAM4 \leq 100$

SHADE IMAGE: BAD PARAMETERS

The parameters to the shade image routine do not satisfy the condition $PARAM1 < PARAM2$.

A1.5 MODIFYING AND ADDING ALGORITHMS TO BLIPS.

There are two stages to adding a new algorithm to the BLIPS package. The first is the calling and specification of the new procedure and the second is the specification of its parameters.

STAGE 1.

A new switch label should be added to the routine EXECUTE COMMAND in the control routine. At this switch label a call of the new procedure should be made specifying the user given parameters which will be in to array PLIST, for integer parameters, and SPLIST for string parameters.

e.g. NEWALG(IMAGE,NLINE,LLINE,PLIST(1),PLIST(2),SPLIST(1))

A specification of this external procedure should also be given.
e.g. EXTERNALROUTINESPEC newalg (INTEGERARRAYNAME image, C
INTEGER n1, l1, white, black, STRING(15) s)

STAGE 2.

Four constant arrays in the control routine must also be modified to allow the checking of the new command and the user supplied parameters.

- 1) Array CLIST contains the command verbs with spaces removed. The new command name should be added to this array.
- 2) Array CTYPE contains a description of the valid parameters for each command. The description in CTYPE is of the form.
Bits 2**0 to 2**2 contain the number of parameters.
Bits 2**3 to 2**7 contain the type of each parameter. A bit set in this range specifies a string parameter. A bit not set in this range specifies an integer parameter.
- 3) Array ITYPE contains a flag for each command to specify whether the current image should be available or not. This should be set to the required value.
- 4) Array SNUM contains the switch number of the new command in the routine EXECUTE COMMAND. This should be set to the new value.

The control routine should then be recompiled and linked with the new algorithm. The new algorithm should now be available from the package.

A2. BIBLIOGRAPHY.

- AND70 Andrews, H.C.
 Computer Techniques in Image Analysis.
 Academic Press 1970.
- AND71 Anderson, G.B. and Huang, T.S.
 Picture Bandwidth Compression by Piecewise Fourier
 Transformations.
 I.E.E.E. Trans. Comm. Technol. COM-19. 1971.
- AND72 Andrews, H.C.
 Some Unitary Transformations in Pattern Recognition and
 Image Processing.
 in 'Information Processing 71', North Holland Press. 1972.
- ATT54 Attneave, F.
 Some Informational Aspects of Visual Perception.
 Psychology Review. 61. 1954.
- BAL40 Baldwin, M.W.
 The Subjective Sharpness of Simulated Television Images.
 Bell Systems Technical Journal. 19. 563. 1940.
- BAR70 Barrow, H.G. and Popplestone, R.J.
 Relational Descriptors in Picture Processing.
 Machine Intelligence 6. Edinburgh University Press. 1970.

- BEN70 Bentley, R.E., Milaw, J. and Cope, D.W.
The Use of Computer Graphic Input and Output Devices
Attached to a Small Computer for Planning Radiotherapy
Treatment.
International Symposium:- Computer Graphics 70.
- BRA68 Bracewell, R.N.
The Fourier Transform and its Applications.
McGraw-Hill. New York. 1968.
- BRI70 Brice, C.R. and Fennema, C.L.
Scene Analysis Using Regions.
Artificial Intelligence 1. 1970.
- BUD72 Budrikis, Z.L.
Visual Fidelity Criteria and Modelling.
Proc. I.E.E.E. 60. 1972.
- CHI74 Chien, Y.P. and King-Sun Fu.
Recognition of X-Ray Picture Patterns.
I.E.E.E. Trans. on Systems, Man and Cybernetics.
SMC-4. No.2 1974.
- CHO72 Chow, C.K. and Kaneko, T.
Automatic Boundary Detection of the Left Ventricle From
Cineangiograms.
Computers in Biomedical Research. 5. 1972.
- COR70 Cornsweet, T.N.
Visual Perception.
Academic Press. 1970.
- DOD68 Dodd and Wallace.
The Venous Diameter Ratio in the Radiographic Diagnosis
of Breast Cancer.
Radiology. 90. May. 1968.

- DOY60 Doyle, W.
Recognition of Sloppy, Hand-Printed Characters.
Proc. W.J.C.C. May. 1960.
- DUD72 Duda, R.O. and Hart, P.E.
Use of the Hough Transformation to Detect Lines and
Curves in Pictures.
C.A.C.M. 15. Jan. 1972.
- DUD73 Duda, R.O. and Hart, P.E.
Pattern Recognition and Scene Analysis.
Wiley Interscience. 1973.
- DUF73 Duff, H.J.B., Watson, D.M., Fountain, T.J. and Shaw, G.K.
A Cellular Logic Array for Image Processing.
Pattern Recognition. 5. 1973.
- DWY72 Dwyer, S.J., Harlow, C.A., Ausherman, D.A. and Lodwick, G.
Computer Diagnosis of Radiographic Images.
A.F.I.P.S. Conf. Proceedings S.J.C.C. 40. 1972.
- EVA69 Evans, D.M.D. (Ed.)
Cytology Automation.
Livingstone Press Edinburgh. 1969.
- FIS69 Fischler, M.A.
Machine Perception and Description of Pictorial Data.
Proc. Int. Joint Conf. on Art. Int. May. 1969.
- FRE61a Freeman, H.
On The Encoding of Arbitrary Geometric Configurations.
I.R.E. Trans. on Elec. Comp. EC-10. June. 1961.
- FRE61b Freeman, H.
Techniques for the Digital Computer Analysis of Chain
Encoded Arbitrary Plane Curves.
Proc. Natl. Elec. Conf. 18. 1961.

- FRE72 Freundlich, I.M.
 Thermography.
 New England Journal of Medicine. 287. 1972.
- FU67 Fu, K.S., Chien, Y.T. and Cardillo, G.P.
 A Dynamic Programming Approach to Sequential Pattern
 Recognition.
 I.E.E.E. Trans. on Elec. Comput. EC-16. Dec. 1967.
- FU70 Fu, K.S. and Swain, P.H.
 On Syntactic Pattern Recognition.
 In 'Software Engineering' Ed. Tou, J.
 Academic Press. 1970.
- GOL69 Goldwyn, R.M., Loh, L. and Siegel, J.H.
 The Analysis of Physiologic Abnormalities in the
 Critically Ill Using a Time Sharing System for the
 Conversational Manipulation of a Large Data Bank.
 Proc. Annual Princeton Conf. on Information Sciences
 and Systems. 1969.
- GOL72 Goldstein, A.J., Harmon, L.D. and Lesk, A.B.
 Man Machine Interaction in Human Face Identification.
 Bell Syst. Tech. Journal. 51. 1972.
- GR071 Groner, G.F., Clark, A.L., Berman, R.A. and Deland, E.L.
 BIOMOD: An Interactive Graphics System for Modelling.
 A.F.I.P.S. Press F.J.C.C. 1971.
- GUI68 Guignon, J.E. and Kline, R.M.
 Development of an On-Line Image Processing System.
 Comp. Syst. Lab. Washington Univ. Tech. Rep. TR5. Feb. 1968.
- HAC70 Hackett, R.
 Diagnostic Revolution with IR Camera.
 Electronic Age. Col 1. May. 1970.

- HAL71 Hall, E.L., Kruger, R.P., Dwyer, S.J., Hall, D.L.,
 McLaren, R.W. and Lodwick, G.S.
 A Survey of Preprocessing and Feature Extraction
 Techniques For Radiographic Images.
 I.E.E.E. Trans. on Comp. C-20. 1971.
- HAR69 Haralick, R.M. and Kelly, G.L.
 Pattern Recognition with Measurement Space and Spatial
 Clustering for Multiple Images.
 Proc. I.E.E.E. Vol 57. April. 1969.
- HAR75 Harlow, C.A., Henderson, S.E., Rayfield, D.A.,
 Johnston, R.J. and Dwyer, S.J.
 Automated Inspection of Electronic Assemblies.
 Computer. April. 1975.
- HIG61 Highleyman, W.H.
 An Analog Method for Character Recognition.
 I.R.E. Trans. on Elec. Comp. EC-10. Sep. 1961.
- HIL69 Hilditch, C.J.
 Linear Skeletons From Square Cupboards.
 Machine Intelligence. 4. Edinburgh University Press. 1969.
- HOL65 Holmes, W.S.
 Optical-Electronic Spatial Filtering for Pattern
 Recognition.
 In 'Optical and Electro-Optical Information Processing 6'
 Ed. Tippett, J.T. M.I.T. Press. 1965.
- HSC72 Health Science Computing Facility.
 Annual Report 1972.
 University of California. Los Angeles.

- HUB62 Hubel, D.H. and Wiesel, T.N.
Receptive Fields, Binocular Interaction and **Functional**
Architecture in the Cat's Visual Cortex.
Journal of Physiology. 160. 1962.
- HUB63 Hubel, D.H.
The Visual Cortex of the Brain.
Scientific American. 209. Nov. 1963.
- HUE71 Hueckel, M.H.
An Operator Which Locates Edges in Digitised Pictures.
J.A.C.M. Vol. 18. Jan. 1971.
- HUE73 Hueckel, M.H.
A Local Visual Operator Which Recognises Edges and Lines.
J.A.C.M. Vol. 20. Oct. 1973.
- ISA72 Isard, H.J., Becker, W., Shilo, R. and Ostrum, B.J.
Breast Thermography After Four Years and 10000 Studies.
American Journal of Roentgenology. 115. 1972.
- JAI74 Jain, A.K. and Angel, E.
Image Restoration, Modelling and Reduction of
Dimensionality.
I.E.E.E. Trans. on Comput. C-23. 1974.
- JOH70 Johnston, E.G. and Rosenfeld, A.
Geometrical Operations on Digitised Pictures.
In 'Picture Processing and Psychopictorics'
Eds. Lipkin, B.S. and Rosenfeld, A. Academic Press. 1970.
- KAN72 Kanal, L.N.
Interactive Pattern Analysis and Classification Systems:
A Survey and Commentary.
Proc. of I.E.E.E. Vol. 60. No. 10. Oct. 1972.

- KAN73 Kanade, T.
Picture Processing System by Computer Complex and
Recognition of Human Faces.
Ph.D. Thesis Dept. of Information Science.
Kyoto University Japan. 1973.
- KEL71 Kelly, M.D.
Edge Detection in Pictures Using Planning.
Machine Intelligence 6. Ed. Meltzer, B. and Michie, D.
Edinburgh University Press. 1971.
- KIM75 Kimme, C., Ballard, D. and Sklansky, J.
Finding Circles by an Array of Accumulators.
C.A.C.M. Vol. 18. No. 2. 1975.
- KIR71 Kirsch, R.A.
Computer Determination of the Constituent Structure of
Biological Images.
Computers in Biomedical Research. 4. 1971.
- LAI77 Laing, W.A.
Results of a Selection of Image Analysis
Algorithms Applied to 70 Breast Thermograms.
Personal Copies.
- LAW63 Lawson, R.W. and Chugtai, M.S.
Breast Cancer and Body Temperature.
Canadian Medical Journal Association. Vol. 88. Jan. 1963.
- LED69 Ledley, R.S. and Rotolo, L.S.
Application of Pattern Recognition to Biomedical Problems.
In 'Automatic Interpretation and Classification of Images'
A NATO Advanced Studies Institute. Academic Press. 1969.

- LEG73 The Aliasing Problems in 2-Dimensional Sampled Imagery.
In 'Perception of Display Information'. Ed. Liberman, L.M.
Plenum Press. New York. 1973.
- LEH72 Lehr, J.T., Lodwick, G.S., Garrotto, L.J. Manson, D.J. and
Nicholson, B.F.
MARS: Missouri Automated Radiology System.
A.F.I.P.S. Conf. Proc. S.J.C.C. Vol. 40. 1972.
- LEN70 Lendaris, G.G. and Stanley, S.L.
Diffraction-Pattern Sampling For Automatic Pattern
Recognition.
Proc. of I.E.E.E. Vol. 58. Feb. 1970.
- LIL69 Lilienfeld, A.M., Barnes, J.M., Barnes, R.B.,
Brasfield, R., Connel, J.F., Diamond, E.,
Gershon-Cohen, J., Haberman, J., Isaro, H.J.,
Lane, W.Z., Lattes, R., Miller, J., Seaman, W.
and Sherman, A.
An Evaluation of Thermography in The Detection of Breast
Cancer. A Cooperative Pilot Study.
Cancer 24. 1969.
- LIN66 Linfoot, E.H.
Fourier Methods in Optical Image Evaluation.
Focal Press. 1966.
- LOD63a Lodwick, G.S., Hann, C.C., Smith, W.E., Keller, A.F. and
Robertson, E.D.
Computer Diagnosis of Primary Bone Tumours.
Radiology. Vol. 80. Feb. 1963.

- LOD63b Lodwick, G.S., Keats, T.E. and Dorst, J.P.
The Coding of Roentogen Images For Computer Analysis As
Applied to Lung Cancer.
Radiology. Vol. 81. No. 2. Aug. 1963.
- MAR72 Martelli, A.
Edge Detection Using Heuristic Search Methods.
Computer Graphics and Image Processing. 1. 1972.
- MER73 Merrill, R.D.
Representation of Contours and Regions For Efficient
Computer Search.
C.A.C.M. Vol. 16. No. 2. Feb. 1973.
- MON71 Montanari, U.
On The Optimum Detection of Curves in Noisy Pictures.
C.A.C.M. Vol. 14. 1971.
- MUN68 Munson, J.H.
Experiments in the Recognition of Hand Printed Text:
Part 1. Character Recognition.
Proc. F.J.C.C. Dec. 1968.
- MYL71 Mylopoulds, J.P. and Pavlidis, T.
On the Topological Properties of Quantised Spaces:
Part 1. Connectivity and Order of Connectivity.
J.A.C.M. Vol. 18. 1971.
- OHA72 O'Handley, D.A. and Green, W.B.
Recent Develpements in Digital Image Processing at the
Image Processing Laboratory at the Jet Propulsion
Laboratory.
Proc. of I.E.E.E. 60. 1972.

- OSB71 Osborne, P.J.H., Watson, J.L. and Gore, W.G.
The A.W.R.E. Scanning Thermometer.
Engineering Review. April 1971.
- PAT69 Patrick, E.A.
'INTERSPACE': Interactive System For Pattern Analysis,
Classification and Enhancement.
I.E.E.E. Comp. and Comm. Conf. Sep. 1969.
- PET62 Peterson, D.P. and Middleton, D.
Sampling and Reconstruction of Wave-Number-Limited
Functions in n-Dimensional Euclidean Spaces.
Information and Control. 5. 1962.
- PIP68 Pipberger, H., Schneiderman, M.A. and Klingeman, J.D.
The 'Love-at-First-Sight' Effect in Research.
Circulation. Vol. 38. Nov. 1968.
- PRA69 Pratt, W.K., Kane, J. and Andrews, H.C.
Hadamard Transform Image Coding.
Proc. of I.E.E.E. 57. 1969.
- RAM69 Ramsey, D.M.
Image Processing in Biological Science.
University of California Press 1969.
- REG67 The Registrar General's Statistical Review.
For The Two Years 1966-1967. Supplement on Cancer.
H.M.S.O.
- ROB65 Roberts, L.G.
Machine Perception of Three Dimensional Solids.
In 'Optical and Electro-Optical Information Processing'.
Ed. Tippett, J.T. M.I.T. Press. 1965.

- ROS66 Rosenfeld, A. and Pfaltz, J.L.
 Sequential Operations in Digital Picture Processing.
 J.A.C.M. Vol. 13. 1966.
- ROS68 Rosenfeld, A. and Pfaltz, J.L.
 Distance Functions on Digital Pictures.
 Pattern Recognition 1. July. 1968.
- ROS69 Rosenfeld, A.
 Picture Processing by Computer. (69).
 Computer Surveys. Vol. 1. 1969.
- ROS72 Rosenfeld, A.
 Picture Processing. (72).
 Computer Graphics and Image Processing. 1. 1972.
- ROS73 Rosenfeld, A.
 Progress in Picture Processing (69-71).
 Computing Surveys. Vol. 5. No. 2. 1973.
- ROS74 Rosenfeld, A.
 Picture Processing. (73).
 Computer Graphics and Image Processing. 3. 1974.
- ROS75 Rosenfeld, A.
 Picture Processing. (74).
 Computer Graphics and Image Processing. 4. 1975.
- SAM68 Sammon, J.W.
 On-Line Pattern Analysis and Recognition System. (OLPARS)
 Rome Air Development Centre Tech. Rep. TR-68-263. 1968
- SC076 Scott, A.
 Personal Communication.

- SHA49 Shannon, C.E.
 The Mathematical Theory of Communication.
 University of Illinois Press. 1949.
- STE75 Stephens, P.D.
 The IMP Language and Compiler.
 Computer Journal. Vol. 17. No. 3. 1975.
- ST072 Stockham, T.G.
 Image Processing in The Context of a Visual Model.
 Proc. I.E.E.E. 60. 1972.
- WHI70 White, J. and Perkins, J.
 Interactive Computer Generated Stereoscopic Displays For
 Biomedical Research.
 International Symposium:- Computer Graphics 70.
- WHI73 Whitfield, H. and Wight, A.S.
 EMAS - The Edinburgh Multi-Access System.
 Computer Journal. Vol. 16. No. 4. 1973.
- WIL62 Wilks, S.S.
 Mathematical Statistics.
 John Wiley. 1962.
- WIL70 Wilton-Davies, C.C.
 Small Computer Graphics in The Physiology Laboratory.
 International Symposium:- Computer Graphics 70.
- W0064 Wood, D.
 Thermography in The Diagnosis of Cerebrovascular Disease.
 Radiology. 83. Sep. 1964.
- ZAH69 Zahn, C.T.
 A Formal Description of Two Dimensional Patterns.
 Proc. Int. Joint Conf. on Art. Int. May. 1969.

ZUS70 Zusne, L.
Visual Perception of Form.
Academic Press. 1970.